TIME-VARYING FORWARD BIAS AND THE VOLATILITY OF RISK PREMIUM: A MONETARY EXPLANATION

Juan Ángel Lafuente (*)
Universidad Jaume I
Unidad Predepartamental de Finanzas y Contabilidad
Campus del Riu Sec, 12080, Castellón

Jesús Ruiz (*)
Instituto Complutense de Análisis Económico (ICAE)
Universidad Complutense
Campus de Somosaguas, 28223 Madrid

ABSTRACT

Forward exchange rate unbiassedness is rejected for international exchange markets. This paper proposes a stochastic general equilibrium model which generates substantial variability in the magnitude of predictable excess returns. Simulation exercises suggest that high persistency in the monetary policy produces greater bias in the estimated slope coefficient in the regression of the change in the logarithm of the spot exchange rate on the forward premium. Also, our model suggest that the nature of the transmission between monetary shocks can explain the excess return puzzle. Empirical evidence for the US-UK exchange rate according to our theoretical results is provided.

RESUMEN

La insesgadez del tipo de cambio forward es rechazada para los mercados cambiarios internacionales. Este trabajo propone un modelo de equilibrio general dinámico y estocástico que genera variabilidad suficiente en las magnitudes de los excesos de rendimientos predecibles. Los ejercicios de simulación realizados sugieren que una alta persistencia de la política monetaria produce un mayor sesgo en el coeficiente estimado de la pendiente de la regresión entre la primera diferencia del logaritmo del tipo de cambio spot sobre la prima forward. Además, nuestro modelo sugiere que la naturaleza de la transmisión entre shocks monetarios puede explicar la paradoja del exceso de rendimiento. Por último, proporcionamos evidencia empírica de acuerdo con nuestros resultados teóricos para el tipo de cambio entre EEUU y Reino Unido.

(*) Los autores agradecen al profesor Alfonso Novales sus valiosos comentarios y sugerencias. Jesús Ruiz agradece también los comentarios y sugerencias proporcionados por los miembros del CREB así como los medios que han puesto a su disposición durante el tiempo que estuvo invitado como visitante (febrero y marzo de 2002). Agradecemos el apoyo financiero del Ministerio de Educación a través de las becas PB98-0831 y BEC2000-1388-C04-03.

1 Introduction

The puzzle of biasedness on forward exchange rate refers that the estimated slope coefficients in the regression of the change in the logarithm of the spot rate on the forward premium significantly departs from one (see Zhu (2002), Tauchen (2001), Baillie and Bollerslev (2000), Baillie and Ostenberg (2000) and McCallum (1994), among many others). Such discrepancy from the underlying value in the uncovered interest rate parity implies that the forward rate is not an unbiased predictor of the future spot rate, suggesting the possibility of unexploited profit opportunities. Potential explanations of this excess return puzzle generally are assigned to three kind of categories: a) the most popular is that such pattern arises as a consequence of a time-varying risk premia (see Fama (1984)); b) a second explanation relies on the nature of expectations. Under no rational expectations agents do not efficiently use the available information set, incurring in systematic forecasting errors over a significant number of time periods ahead (see Froot and Frankel (1989)); and c) the peso problem, that is, market participants anticipate by rational learning process a future discrete shift in policy that is not performed within the sample period analyzed (see Lewis, (1995)).

Even though a substantial number of studies have addressed the ability of general equilibrium models related to the Lucas (1982) model to explain the forward premium puzzle (see, for example, Hodrick (1989), Macklem (1991), Canova and Marrinan (1993), Bekaert (1994)), they unsuccessfully explained the substantial variability that occur in the magnitude of predictable excess returns. Currently, there is no conclusive theory explaining the behavior of the bias of test for a risk premium in forward exchange rates, and it is yet regarded as one of the most important unresolved puzzle in international finance.

In this paper we develop a theoretical general equilibrium model to explain short and long-run risk premium in forward markets for foreign exchange that, not only provide additional insights about the potential explaining factors of the forward risk premium, but also reproduce the forward premium anomaly under rational expectations. The model takes as benchmark the Dutton's model (1993) which is based on the general equilibrium models of Lucas (1982). Our model extend the first one in three ways: a) we consider two forward (one and two periods) exchange rates as a hedge instruments for spot exchange rate a more realistic approach to real markets in where different time to maturity can be traded. This enriches the analysis because of it would be possible to identify the effect of the time to maturity in the derivative contract on the forward market risk premia, b) it is considered the possibility that domestic and foreign consumptions goods will be complementary or substitutes. Therefore, the model allows to estimate the impact of the nature of consumptions goods. If, for example, dollars are relatively risk, the uncertainty about the future spot exchange should affect differently on the forward risk premia under complementaries or substitutes consumption goods, and c) the weight of each, domestic and foreign, consumption good in the utility function is not necessary the same. Consequently, a broad set of scenarios can be simulated in order to explore for potential explanatory factors of the risk premium.

The solution of the model involves to evaluate expectations of nonlinear expressions. Therefore, numerical solutions are provided. Under the assumption of rational expectations, our solution method allows to solve jointly for both prices and positions in one and two-periods ahead forward contracts. This is an interesting extension relative to the Dutton's solution method.

Simulation exercises are carried out with a variety of parameter values, revealing that, even when the econometric bias behind the regression of the change in the logarithm of the spot rate on the forward premium is taken into account, the model can reproduce the bias for forward exchange rate to predict the future evolution of spot rate. The model suggest what are the key factor generating high volatility for the risk premium is the persistence of the monetary policy. Under a relative high persistence the estimated slopes dramatically decreases below one. Moreover, theoretical results show that the time to maturity of forward contract is negatively correlated with the size of the slope coefficient in the regression, that is, the estimated slopes corresponding to the long time to maturity contract are relatively lower.

In accordance with the theoretical simulations of the model, the paper reports empirical evidence for the US dollar -British pound, not only on the relationship between the correlation of the monetary shocks and the size of estimated slopes, but also on the linkage between the persistence of the monetary policy and the bias for forward exchange rate.

The rest of the paper is organized as follows: section 2 present the model. In section 3 simulations of risk premium are presented and theoretical results about the bias of forward premium are provided. Section 4 refers empirical evidence for the US-UK exchange rate. Finally, section 5 summarizes and makes concluding remarks.

2 The Model

There are two countries with its own currency and a single consumer. In each country the representative firm receives an endowment of a single traded good. The only tradable financial assets are the money forward periods exchange contracts. Also, there is no contingent claims markets, so all possibilities to reduce risk are concerning the forward exchange market, where two maturity contracts are available.

The two consumers own titles to the firms in their respective countries. The timing of the model can be summarized as follows: 1) at the beginning of each period, both firms pay to the respective consumers in its country a dividend equal to all incomes achieved the previous period. Then, the consumer turns in its dividends for a new money, and the old money becomes worthless. This implies that all money will be spent; 2) after receiving the money supply, consumers liquidate their forward contracts traded in foreign exchange in the two previous periods, 3) consumers spend their money on the two goods. Domestic goods must be purchased with its own currency. All transactions take place at equilibrium prices. 4) At the end of each period, consumers make forward contracts to delivery of currency in the next two periods.

Endowments of goods and money supplies are stochastic, and its natural logarithm follow an autoregressive process with a Normal innovation. Let us to denote X_t and M_t for any good endowment or money supply, respectively:

$$\ln X_t = \mu_X(1 - \rho_X) + \rho_X \ln X_{t-1} + \xi_{X;t}\ ; \qquad \xi_{X;t} \sim N\left(0; \sigma_X^2\right), \tag{1}$$

$$\ln X_t^* = \mu_{X^*}(1 - \rho_{X^*}) + \rho_{X^*} \ln X_{t-1}^* + \xi_{X;t}^*\ ; \qquad \xi_{X^*;t} \sim N\left(0; \sigma_{X^*}^2\right), \tag{2}$$

$$\ln M_t = \mu_M(1 - \rho_M) + \rho_M \ln M_{t-1} + \xi_{M;t}\ ; \qquad \xi_{M;t} \sim N\left(0; \sigma_M^2\right), \tag{3}$$

$$\ln M_t^* = \mu_{M^*}(1 - \rho_{M^*}) + \rho_{M^*} \ln M_{t-1}^* + \xi_{M;t}^*\ ; \qquad \xi_{M^*;t} \sim N\left(0; \sigma_{M^*}^2\right), \tag{4}$$

where the asterisk denotes the foreign country. Correlations between any four shocks ($\rho_{MM^*}; \rho_{XX^*}; \rho_{MX}; \rho_{MX^*}; \rho_{M^*X}; \rho_{M^*X^*}$) are initially restricted to be zero.

2.1 The Consumer's problem

The utility function of the home consumer is a CES function:

$$U_t = \frac{1}{1-\gamma}\left[\phi\,(C_{D,t})^{\varepsilon} + (1-\phi)\,(C_{F,t})^{\varepsilon}\right]^{(1-\gamma)/\varepsilon}, \tag{5}$$

where $C_{D,t}$ and $C_{F,t}$ are the consumption levels of domestic and foreign goods at time t, γ is the coefficient of relative risk aversion, and $\frac{1}{1-\varepsilon}$ is the elasticity of substitution, and ϕ is the weight for each consumption good. If ε approaches to zero consumption goods becomes more substitutes, whereas complementary arises when ε approaches to one. The parameter ϕ measures the weight of each consumption good in the utility function. The optimization problem for the home consumer is:

$$\underset{\{C_{D,t};\,C_{F,t}\}}{\text{Max}} \quad E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}\left[\phi\,(C_{D,t})^{\varepsilon} + (1-\phi)\,(C_{F,t})^{\varepsilon}\right]^{(1-\gamma)/\varepsilon}\right] \tag{6}$$

s.t.

$$P_{D,t}C_{D,t} + S_tP_{F,t}C_{F,t} \leq Y_t,$$

$$Y_t = M_t + T_{t-1,1}\frac{S_t - F_{t-1,1}}{F_{t-1,1}} + T_{t-2,2}\frac{S_t - F_{t-2,2}}{F_{t-2,2}},$$

where $P_{D,t}$ and $P_{F,t}$ are the prices of domestic and foreign goods at time t, Y_t is the total income in period t, S_t is the spot exchange rate, $F_{t-1,1}$, $F_{t-2,2}$ are the prices for the two maturity forward contracts available, $T_{t-1,1}$ and $T_{t-2,2}$ are the respective amount of its currency that the home country sold forward in the two previous periods. The money supply (M_t) plus the profits on each forward currency trading in period t equals the total home income. A similar optimization problem can be pointed out for the foreign consumer, that is:

$$\underset{\{C^*_{D,t};\,C^*_{F,t}\}}{\text{Max}} \quad E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}\left[\phi\left(C^*_{D,t}\right)^{\varepsilon} + (1-\phi)\left(C^*_{F,t}\right)^{\varepsilon}\right]^{(1-\gamma)/\varepsilon}\right] \tag{7}$$

s.t.

$$P_{D,t}C^*_{D,t} + S_tP_{F,t}C^*_{F,t} \leq Y^*_tS_t,$$

$$Y^*_t = M^*_t + T^*_{t-1,1}\frac{S_t - F_{t-1,1}}{F_{t-1,1}S_t} + T^*_{t-2,2}\frac{S_t - F_{t-2,2}}{F_{t-2,2}S_t}.$$

2.1.1 Optimal good choices.

In any period t the home consumer chooses levels of $C_{D;t}$ and $C_{F;t}$ that maximize U_t subject to the level of total home income. First order conditions for choice of $C_{D;t}$ and $C_{F;t}$ are:

$$\left[\omega (C_{D;t})^{\rho} + (1-\omega)(C_{F;t})^{\rho}\right]^{\frac{1-\gamma}{\rho}-1} (C_{D;t})^{\rho-1} - \lambda_t P_{D;t} = 0 \ , \qquad (8)$$

$$\left[\omega (C_{D;t})^{\rho} + (1-\omega)(C_{F;t})^{\rho}\right]^{\frac{1-\gamma}{\rho}-1} (C_{F;t})^{\rho-1} - \lambda_t S_t P_{F;t} = 0 \ , \qquad (9)$$

$$Y_t - P_{D;t}C_{D;t} - P_{F;t}S_tC_{F;t} = 0 \ . \qquad (10)$$

From 8 and 9 yields the following relationships:

$$C_{F;t} = \left[\frac{(1-\omega)P_{D;t}}{\omega P_{F;t}S_t}\right]^{\sigma} C_{D;t} \ , \qquad (11)$$

where $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution. Using 11 and the budget constraint, the demand function for the domestic good is the following:

$$C_{D;t} = \frac{Y_t P_{D;t}^{-\sigma}}{P_{D;t}^{1-\sigma} + \left(\frac{1-\omega}{\omega}\right)^{\sigma} (S_t P_{F;t})^{1-\sigma}} \ . \qquad (12)$$

Substituting 12 into equation 11 the next demand function for the foreign good arises:

$$C_{F;t} = \left[\frac{(1-\omega)P_{D;t}}{\omega P_{F;t}S_t}\right]^{\sigma} \frac{Y_t P_{D;t}^{-\sigma}}{P_{D;t}^{1-\sigma} + \left(\frac{1-\omega}{\omega}\right)^{\sigma} (S_t P_{F;t})^{1-\sigma}} \qquad (13)$$

Similar equation to (12) and (13) can be easily found for the foreign country:

$$C^{*}_{F;t} = \left[\frac{(1-\omega)P_{D;t}}{\omega P_{F;t}S_t}\right]^{\sigma} C^{*}_{D;t} \qquad (14)$$

$$C^{*}_{D;t} = \frac{Y^{*}_t S_t P_{D;t}^{-\sigma}}{P_{D;t}^{1-\sigma} + \left(\frac{1-\omega}{\omega}\right)^{\sigma} (S_t P_{F;t})^{1-\sigma}} \qquad (15)$$

Substituting 15 into 14 we obtain the analytical expression for $C^{*}_{F;t}$.

2.1.2 Forward Contracting

As well as the allocation of current resources between the two goods, the home consumer choose in period t the levels of the one and two periods forward contracting, that is $T_{t,1}$ and $T_{t,2}$. The Euler conditions are:

$$E_t\left[\lambda_{t+1}\beta^{t+1}\left(\frac{S_{t+1}-F_{t,1}}{F_{t,1}}\right)\right]=0\;, \tag{16}$$

$$E_t\left[\lambda_{t+2}\beta^{t+2}\left(\frac{S_{t+2}-F_{t,2}}{F_{t,2}}\right)\right]=0\;, \tag{17}$$

where E_t denotes the conditional expectation to the information set available in period t. From 16:

$$E_t\left[\lambda_{t+1}S_{t+1}\right]=E_t\left[\lambda_{t+1}F_{t,1}\right],$$

and taking into account 8 yields:

$$F_{t,1}=\frac{E_t\left[\frac{\partial U_{t+1}}{\partial C_{F,t+1}}\frac{1}{P_{F,t+1}}\right]}{E_t\left[\frac{\partial U_{t+1}}{\partial C_{F,t+1}}\frac{1}{P_{F,t+1}S_{t+1}}\right]}\;. \tag{18}$$

Similar rearranging from 17 when taking into account 8 leads to the following expression for the two-periods forward price:

$$F_{t,2}=\frac{E_t\left[\frac{\partial U_{t+2}}{\partial C_{F,t+2}}\frac{1}{P_{F,t+2}}\right]}{E_t\left[\frac{\partial U_{t+2}}{\partial C_{F,t+2}}\frac{1}{P_{F,t+2}S_{t+2}}\right]}\;. \tag{19}$$

Analogous expressions to (18) and (19) can be obtained when the foreign consumer chooses in period t the levels of the one and two periods forward contracting, that is $T^{*}_{t,1}$ and $T^{*}_{t,2}$:

$$F_{t,1}=\frac{E_t\left\{\frac{\partial U^{*}_{t+1}}{\partial C^{*}_{F,t+1}}\frac{1}{P_{F,t+1}}\right\}}{E_t\left\{\frac{\partial U^{*}_{t+1}}{\partial C^{*}_{F,t+1}}\frac{1}{P_{F,t+1}S_{t+1}}\right\}}\;, \tag{20}$$

$$F_{t,2}=\frac{E_t\left\{\frac{\partial U^{*}_{t+2}}{\partial C^{*}_{F,t+2}}\frac{1}{P_{F,t+2}}\right\}}{E_t\left\{\frac{\partial U^{*}_{t+2}}{\partial C^{*}_{F,t+2}}\frac{1}{P_{F,t+2}S_{t+2}}\right\}}\;. \tag{21}$$

2.2 Market-Clearing

2.2.1 Equilibrium in the Goods Market.

The world constraints on consumptions of the two traded goods in both countries implies that the total endowment of the two goods must be equal the consumption of each good in the respective countries, that is:

$$C_{D,t} + C^{*}_{D,t} = X_{D,t} \quad , \tag{22}$$

$$C_{F,t} + C^{*}_{F,t} = X_{F,t} \quad . \tag{23}$$

Equilibrium prices of the two goods depend on the home and foreign money supplies as well as their total endowment in each country. Taking into account that a) money is worthless after each period and b) each country's good only can be purchased with the country's currency, the following cash-in-advance spending constraints must be hold:

$$P_{D,t} X_{D,t} = M_t \quad , \tag{24}$$

$$P_{F,t} X_{F,t} = M^{*}_t \quad . \tag{25}$$

Since goods endowments $X_{D,t}$ and $X_{F,t}$, and money supplies M_t and M^{*}_t are exogenous, the two above equations determine prices of consumption goods.

The solution of the model requires the evaluation of expectations in equations 18 and 19, in where highly non-linear expressions appear. This avoids the possibility of an analytical solution. Appendix 1 provides detailed explanation about the solution method to obtain simulated equilibrium in spot and forward exchange markets. It allows the joint search of all variables (prices and positions) concerning the forward market. In equilibrium, the following relationships between home and foreign derivative positions holds

$$T_{t-l,l} = -T^{*}_{t-l,l} \quad , \quad l = 1, 2. \tag{26}$$

3 Simulation of forward prices and risk premiums

The equilibrium spot rates can be obtained as follows: using the budget constraints, $P_{D,t}\, C_{D,t} + S_t\, P_{F,t}\, C_{F,t} = Y_t$ and $P_{D,t}\, C^{*}_{D,t} + S_t\, P_{F,t}\, C^{*}_{F,t} = Y^{*}_t S_t$

and equations (11) and (14), we can solve analytically the spot exchange as a function of the exogenous stochastic variables $X_{D,t}, X_{F,t}, M_t, M_t^*$:

$$s_t = \frac{1-\phi}{\phi}\left(\frac{X_{F,t}}{X_{D,t}}\right)\frac{M_t}{M_t^*}. \tag{27}$$

3.1 Definition of Risk Premium

To avoid the implications of Siegel's paradox we use the following definition of the risk premium in the forward market:

$$rp_{t,t+l} = f_{t,l} - E_t(s_{t+l}), \quad l = 1,2. \tag{28}$$

where $E_t(\cdot)$ denotes the mathematical expectation conditioned on the set of all relevant information at time t, s_t is the logarithm of the domestic currency price of foreign currency at time t and $f_{t,l}$ is the logarithm of the forward exchange rate with delivery at time $t+l$.

3.2 Parameter scenarios where the forward premium anomaly arises

3.2.1 Testing the unbiasedness hypothesis

The main objective of the paper is to analyze the parameter set that could reproduce the forward premium bias. The central hypothesis that we analyze in this paper is the Uncovered Interest Rate Parity (UIP) condition, which states that:

$$E_t(\Delta s_{t+l}) = f_{t,l} - s_t = i_t - i_t^*, \tag{29}$$

where E_t denotes the conditional expectation to the information set available on time t; i_t and i_t^* are the interest rates on domestic and foreign deposits, respectively, and Δ denotes the first difference operator, that is, $\Delta s_{t+l} \equiv s_{t+l} - s_{t+l-1}$.

To test for unbiasedness hypothesis, the literature has widely focused on the following regression relating the change in the spot rate to the forward-spot spread:

$$\Delta s_{t+l} = \alpha_l + \beta_l(f_{t,l} - s_t) + u_{t+l,l}, \tag{30}$$

The estimation of equation (30) tries to test the ability of the forward-spot differential to forecast the direction of change in spot rate. Regardless the sampling frequency, the UIP condition implies that $\alpha_l = 0$ and $\beta_l = 1$. However, empirical evidence has widely reported on estimated slopes that turn out to be below than one or even negatives[1]. This finding not only reject the UIP condition, but also is contradictory with either form of the expectations hypothesis.

The analytical expression for the OLS estimation of β_l is:

$$\beta^{ols} = \frac{Cov\left(f_{t;l} - s_t; s_{t+l} - s_t\right)}{Var\left(f_{t;l} - s_t\right)}, \tag{31}$$

where $Var(\cdot)$ refers to variance, and $Cov(\cdot)$ denotes the covariance. As pointed out in Engel (1996), if the estimator is consistent, under rational expectations it follows that:

$$p\lim\left(\beta^{ols}\right) = 1 - \beta_{rp} \tag{32}$$

where $\beta_{rp} = \frac{Cov\left(E_t(s_{t+l}) - s_t;\, f_{t;l} - E_t(s_{t+l})\right) + Var\left(f_{t;l} - E_t(s_{t+l})\right)}{Var\left(f_{t;l} - s_t\right)}$. From this expression it can be observed that low values of β^{ols} can be explained under rational expectations if $Var\left(f_{t;l} - E_t(s_{t+l})\right)$ is enough large. The risk premium is widely considered the most likely source of the puzzle, but taking into account the regression results reported in the literature the required volatility are far larger than most researchers would accept. One of the major task in the literature concerns to explain why the risk premium has such a large variance. Our model provide some insights about this issue.

3.2.2 Theoretical Results

In all numerical simulations the discount factor β and the relative risk aversion γ are constant and equal to 0.99 and 1.50, respectively[2]. We consider a variety of scenarios than can be summarized as follows: a) we focus the analysis on the effects of the monetary policy (we leave further work the analysis of the effects of real shocks on risk premia in forward markets for

[1] A recent survey can be found in Engel (1996).

[2] Parameter values inside the interval $[0.90; 0.99]$ and $[1.10; 5.00]$ for β and γ, lead to similar results to those reported in the paper.

foreign exchange). Therefore, only a uncertainty source is considered: monetary shocks. This way we consider either one or two shocks; b) we distinguish between situations in where there is no persistence in the shocks of both countries from other ones in where only the home country have persistence in the monetary shock[3]. The nature of the interaction between monetary policies is also examined. When two shocks are considered we allow for three possibilities: uncorrelated, positive and negatively correlated monetary shocks. The considered absolute value for the correlation coefficient between domestic and foreign shocks is 0.9. To summarize the theoretical results from estimating equation 30 using simulated spot and forward exchange rates with $\varepsilon = 1$[4], Table 1 reports the volatility of the forward premium and Figures 1 to 10 (Appendix 3) depict the estimated slopes as a function of the correlation between monetary shock and the persistence of the monetary policy when only a monetary shock is considered. Also we provide the confidence intervals at the 5% significance level based on the simulated distribution of slopes with one hundred of theoretical observations. Several interesting questions emerge from this information set:

1. The estimated slopes are generally lower than one, a consistent finding with expression 30. This means that $\beta_{rp} > 0$. This finding has been documented in many empirical studies (see, for example, Bilson (1981), Fama (1984), Bekaert and Hodrick (1993), Backus et al. (1993) and Mark et al. (1993)).

2. There is a negative relationship between the estimated slope coefficient and the time to maturity. In the long-run the forward bias is greater than in short-run, reflecting a higher uncertainty in the futures evolution of spot rates,

3. A relative higher persistence in the monetary policy produces lower estimated value for the slope. This finding is consistent with those reported in Baillie and Bollerslev (2000). Those authors simulate forward premiums. according to a highly stylized UIP-FIGARCH model (Fractionally Integrated GARCH model), showing that a long memory

[3] Under no correlation between monetary shocks this situation can be interpreted as the home country behaves as a leader since it can update the forecasting of money supply. The considered autoregressive parameter is 0.9.

[4] Similar results are found with $\varepsilon = 0$, which are available from the authors upon request.

in the forward premium produces wide dispersion in the slope coe¢-cients. Tauchen (2001) simulates the sampling distribution of the slope coe¢cient in equation (30), showing that such to be the case when spot rates are generated with a near to non-stationary AR(1) process. This is not surprising when equation (27) is observed. Under high persistence in the monetary policy of the domestic country, spot rate is very autocorrelated, and consequently the forward premium should have high persistence. The negative relationship is clearer when.

4. More interestingly, our model suggest that under a relative high persistence in the domestic/foreign monetary policy the volatility of the forward premium is greater. From table 1, it can be observed that the volatility under persistence is above ...ve times the volatility that corresponds to the case where monetary policy forecast can not be updated using current information.

5. Also, the transmission of the monetary policy e¤ects between both countries appears to be a signi...cant factor to explain departures from the UIP. Under a relative high persistence, the estimated slope show higher discrepancy with the unitary value when monetary shocks are positively correlated. Indeed the maximum median anomaly for all simulations appears when shocks are positively correlated and the domestic monetary policy is very persistent. This a realistic scenario for most of empirical studies that analyses the exchange rate between US and other country, which generally takes as a benchmark the Fed´s monetary policy. Deviations from the UIP condition are negligible regardless the correlation between the monetary shocks only under no persistence in the monetary policy of both countries.

But, what about the ability of the model to generate bias for forward exchange rate?. To answer this question Figures 11 to 20 depict the sum of the asymptotic bias plus the median estimated slopes coe¢cients and their corresponding con...dence intervals at the 5% signi...cance level, again using one hundred of theoretical observations. Those graphs show two relevant aspects:

1. once we have ...ltered the econometric bias a discrepancy with the unitary value remains, revealing that the theoretical model can generate a bias for forward exchange rate.

2. the relative persistence in the monetary policy appears as the key factors behind the forward unbiasedness. It can be observed that the con…dence intervals are larger enough under a relative high persistence, suggesting a higher variability in the potential estimated slopes. In particular, when the two countries apply persistence in the monetary policy the con…dence intervals are much less informative. Moreover, under such scenario the correlation between monetary shocks is an additional factor that explain the forward bias, revealing a higher median deviation from one when shocks are positively rather than negatively correlated (see Figures 19 and 20). Also in this case the con…dence intervals are less informative than under no

In summary, our model suggest that the anomaly should appear when one country act as a leader when monetary policy is implemented and a high persistence is applied. Such is the case in most of empirical analysis that concerns the dollar exchange rate. In the next section we provide empirical evidence about this.

4 Empirical evidence. The US dollar-British pound exchage rate

In this section we provide empirical evidence focusing not only on the relationship between the transmission of monetary shocks and slope coe¢cients, but also on the link between the monetary persistence and the bias for the US dollar-British puund forward exchange rate. The considered time to maturity is one month and the sample period covers from December, 1986 to November 2001.

The model predict a negative relationship between the estimated slopes and the correlation between monetary shocks. Figure 21 show the XY plot of the rolling correlation between the M1 cyclical components[5] of US and UK and the rolling slopes using one month time to maturity US-UK forward exchange rate for the already referred sample period. The window size to compute the rolling statistics corresponds to …ve years. The US average rolling persistence in this period was 0.77. Clearly, and according with our theoretical results, a negative relationship arises. To quantitative account for this statement, we perform the following regression:

[5] The Hodrick-Prescott …lter is used to detrend the monetary aggregate.

$$\beta_t = \delta_0 + \delta_1 \rho_{MM^*;t} + u_{l;t} \quad l = 1; 2 \tag{33}$$

where β_t denotes the actual rolling slope and $\rho_{MM^*;t}$ is the correlation coefficient between the cyclical M1 components. The fitted line is: $\hat{\beta}_{l;t} = 0.88\,(0.09) - 2.39\,(0.21)\,\rho_{MM^*;t}$ where standard errors are in parentheses. The R-squared becomes 0.42.

Also, the model suggests a negative relationship between the estimated slopes coefficients and the persistence of the monetary policy. Figure 22 depict the rolling persistence of the UK monetary policy and the corresponding rolling slopes using a five years moving window over the above referred sample. It can be observed that in three subsamples a negative relationship appears, suggesting that additional factors are affecting the forward bias along the overall sample.

5 Summary and concluding remarks

In this paper we examine the bias of tests for a risk premium in forward exchange rates which refers to significant discrepancies with the unitary value in the estimated slope coefficients from regressions of the change in the logarithm of the spot rate on the forward premium. We perform a theoretical analysis by extending the dynamic and stochastic general equilibrium model with goods endowment proposed in Dutton (1993). Our contribution is the introduction of a two-period forward contract in the derivative market. Also, a solution method under rational expectations is provided.

Our main objective is to explore the effects of the monetary policy and their interactions between the domestic and foreign country on the behavior of the risk premium in order to explain the inconsistency with the UIP condition. Our simulations results suggest that a high persistence in the domestic monetary policy produces greater volatility in the forward premium, and consequently the estimated slope coefficients show greater deviations from one. Moreover, the nature of the transmission between monetary shocks is a potential explaining factor for excess return puzzle. Under persistence, the estimated slopes dramatically decrease below one when monetary shocks are positively correlated. Finally, we find that the time to maturity of the derivative contract is positively related with the bias of risk premium in forward exchange rates. The UIP condition only holds in the absence of persistence when monetary shocks are uncorrelated. or negatively correlated. However, this is an unlikely scenario for most of developed economies.

The paper provides empirical evidence for the US dollar-British pound exchange rate. In accordance with our theoretical results, a negative relationship between the forward bias and the UK monetary persistence is observed along three di¤erent subsamples from December, 1986 to November 2001. Moreover, a negative relationship between the forward bias and the correlation between monetary shocks arises during the overall sample, where a high persistence in the US monetary policy is detected.

While the focus of this paper is the e¤ect of the monetary policy, a similar analysis can be made taking into account the presence of both monetary and real shocks. We leave further work under such scenarios for further research.

References

[1] Backus, D., Gregory A. and C. Telmer (1993), Accounting for forward rates in markets for foreign currency, Journal of Finance 48, 1887-1908.

[2] Baillie, R.T. and T. Bollerslev (2000), The forward premium anomaly is not as bad as you think, Journal of International Money and Finance 19, 471-488.

[3] Baillie, R.T. and W. P. Ostenberg, Deviations from daily uncovered interest rate parity and the role of intervention, Journal of International Financial Markets, Institutions and Money 10, 363-379.

[4] Bilson, J. (1981), The "speculative e¢ciency" hypothesis, Journal of Business 54, 435-452.

[5] Bekaert, G.(1994), Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model, Journal of International Economics 36, 29-52.

[6] Bekaert, G. and R. J. Hodrick (1993), On biases in the measurement of foreign exchange risk premiums, Journal of International Money and Finance 12, 115-138.

[7] Canova F. and J. Marrinan (1993), Pro…ts, risk and uncertainty in foreign exchange markets, Journal of Monetary Economics 32, 259-286.

[8] Dutton, J. (1993), Real and monetary shocks and risk premia in forward markets for foreign exchange, Journal of Money Credit and Banking 25(4), 731-754.

[9] Engle, C. (1996), The forward discount anomaly and the risk premium: A survey of recent evidence, Journal of Empirical Finance 3, 123-192.

[10] Fama, E.F. (1984), Forward and spot exchange rates, Journal of Monetary Economics 14, 319-338.

[11] Froot, K.A. and J.A. Frankel (1989), Forward discount bias: is it an exchange risk premium, The Quaterly Journal of Economics 104, 139-161.

[12] Hodrick, R.J. (1989), Risk, uncertainty and exchange rates, Journal of Monetary Economics 23, 433-459.

[13] Lewis, K.K. (1995). Puzzles in international …nancial markets. In: Grossman, G. and K. Rogao¤ (Eds.). Handbook of International Economics, vol. 3. North Holland, Amsterdam, pp. 1913-1971.

[14] Lucas, Robert (1982), Interest rates and currency prices in a two-country world, Journal of Monetary Economics 10, 335-59.

[15] Macklem, R.T. (1991), Forward exchange rates in arti…cial economies, Journal of International Money and Finance 10, 365-391.

[16] Mark, N., Wu Y. and W. Hai (1993), Understanding spot and forward exchange rate regressions. Ohio State University, Columbus, OH.

[17] McCallum, B.T. (1994), A reconsideration of the uncovered interest rate parity relationship, Journal of Monetary Economics 33, 105-391.

[18] Tauchen, G. (2001), The bias of tests for a risk premium in forward exchange rates, Journal of Empirical Finance 8, 695-704.

[19] Zhu, Z. (2002), Time-varying forward bias and the expected excess return, Journal of International Financial Markets, Institutions and Money 12, 119-137.

Appendix 1. Solution Method

This appendix contains the step that we use in the solution method. As we pointed out in Section 3, the problem concerning the home and foreign consumer is highly non-linear, not allowing to achieve an analytical solution. Therefore a numerical approach must be used.

After providing numerical values for the structural parameters involved in the theoretical economy, that is, $\{\beta; \gamma; \phi; \varepsilon; \sigma_X; \sigma_{X^*}; \sigma_M; \sigma_{M^*}; \mu_X; \mu_{X^*}; \mu_M; \mu_{M^*}; \rho_X; \rho_{X^*}; \rho_M; \rho_{M^*}\}$, the next stages are:

1. We obtain one hundred realizations for the stochastic variables $X_{D;t}$; $X_{F;t}$; M_t; M_t^* in each time period $t = 1; \ldots 100$.

2. One hundred realizations of both home and foreign prices of the consumption goods are computed according to equations (24) and (25), in each time period. Let us to denote this numerical set as $\{(P_{D;t;i}; P_{F;t;i}); i; t = 1; \ldots 100\}$, where i and t denote the realization and the time period, respectively.

3. Similar numerical set to the previous one for P_D and P_F is computed for the spot exchange rate using equation (27), that is, $\{S_{t;i}\ i; t = 1; \ldots 100\}$.

Computation of the forward prices and derivative positions for the one and two period ahead traded contracts $[F_{t;1}; F_{t;2}; T_{t;1}T_{t;2}]$. From equations (11) and (14), substituting into equations (18) and (19) the following expressions can be obtained:

$$
\begin{aligned}
F_{t;1} &= \frac{E_t\left[C_{D;t+1}^{\varepsilon-1}\left(\phi C_{D;t+1}^{\varepsilon} + (1-\phi)C_{F;t+1}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+1}}\right]}{E_t\left[C_{D;t+1}^{\varepsilon-1}\left(\phi C_{D;t+1}^{\varepsilon} + (1-\phi)C_{F;t+1}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+1}S_{t+1}}\right]} \\
&= \frac{E_t\left[W_{D;t+1}\right]}{E_t\left[W_{D;t+1}/S_{t+1}\right]}\ , \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;2} &= \frac{E_t\left[C_{D;t+2}^{\varepsilon-1}\left(\phi C_{D;t+2}^{\varepsilon}+(1-\phi)C_{F;t+2}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+2}}\right]}{E_t\left[C_{D;t+2}^{\varepsilon-1}\left(\phi C_{D;t+2}^{\varepsilon}+(1-\phi)C_{F;t+2}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+2}S_{t+2}}\right]} \\
&= \frac{E_t\left[W_{D;t+2}\right]}{E_t\left[W_{D;t+2}/S_{t+2}\right]}, \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;1} &= \frac{E_t\left[C_{F;t+1}^{*\,\varepsilon-1}\left(\phi C_{D;t+1}^{*\,\varepsilon}+(1-\phi)C_{F;t+1}^{*\,\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{F;t+1}}\right]}{E_t\left[C_{F;t+1}^{*\,\varepsilon-1}\left(\phi C_{D;t+1}^{*\,\varepsilon}+(1-\phi)C_{F;t+1}^{*\,\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{F;t+1}S_{t+1}}\right]} \\
&= \frac{E_t\left[W_{F;t+1}\right]}{E_t\left[W_{F;t+1}/S_{t+1}\right]}, \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;2} &= \frac{E_t\left[C_{F;t+2}^{*\,\varepsilon-1}\left(\phi C_{D;t+2}^{*\,\varepsilon}+(1-\phi)C_{F;t+2}^{*\,\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{F;t+2}}\right]}{E_t\left[C_{F;t+2}^{*\,\varepsilon-1}\left(\phi C_{D;t+2}^{*\,\varepsilon}+(1-\phi)C_{F;t+2}^{*\,\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{F;t+2}S_{t+2}}\right]} \\
&= \frac{E_t\left[W_{F;t+2}\right]}{E_t\left[W_{F;t+2}/S_{t+2}\right]}. \qquad (37)
\end{aligned}
$$

We solve jointly $F_{t;1}$, $F_{t;2}$, $T_{t;1}$ and $T_{t;2}$ by searching values that satisfy the following approximations of the equations (34) to (37):

$$
\begin{aligned}
F_{t;1} &= \frac{\sum_{i=1}^{N}\left[C_{D;t+1;i}^{\varepsilon-1}\left(\phi C_{D;t+1;i}^{\varepsilon}+(1-\phi)C_{F;t+1;i}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+1;i}}\right]}{\sum_{i=1}^{N}\left[C_{D;t+1;i}^{\varepsilon-1}\left(\phi C_{D;t+1;i}^{\varepsilon}+(1-\phi)C_{F;t+1;i}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+1;i}\,S_{t+1;i}}\right]} \\
&= \frac{\sum_{i=1}^{N}\left[W_{D;t+1}\right]}{\sum_{i=1}^{N}\left[W_{D;t+1}/S_{t+1}\right]}, \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;2} &= \frac{\sum_{i=1}^{N}\left[C_{D;t+2;i}^{\varepsilon-1}\left(\phi C_{D;t+2}^{\varepsilon}+(1-\phi)C_{F;t+2;i}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+2;i}}\right]}{\sum_{i=1}^{N}\left[C_{D;t+2;i}^{\varepsilon-1}\left(\phi C_{D;t+2;i}^{\varepsilon}+(1-\phi)C_{F;t+2;i}^{\varepsilon}\right)^{\frac{1-\gamma}{\varepsilon}-1}\frac{1}{P_{D;t+2;i}S_{t+2;i}}\right]} \\
&= \frac{\sum_{i=1}^{N}\left[W_{D;t+2;i}\right]}{\sum_{i=1}^{N}\left[W_{D;t+2;i}/S_{t+2;i}\right]}, \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;1} &= \frac{\sum_{i=1}^{N}\left[C^{*\,\epsilon-1}_{F;t+1;i} \left(\psi C^{*\,\epsilon}_{D;t+1;i} + (1-\psi) C^{*\,\epsilon}_{F;t+1}\right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{F;t+1;i}}\right]}{\sum_{i=1}^{N}\left[C^{*\,\epsilon-1}_{F;t+1;i} \left(\psi C^{*\,\epsilon}_{D;t+1;i} + (1-\psi) C^{*\,\epsilon}_{F;t+1;i}\right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{F;t+1;i}S_{t+1;i}}\right]} \\
&= \frac{\sum_{i=1}^{N}[W_{F;t+1;i}]}{\sum_{i=1}^{N}[W_{F;t+1;i}/S_{t+1;i}]}, \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
F_{t;2} &= \frac{\sum_{i=1}^{N}\left[C^{*\,\epsilon-1}_{F;t+2;i} \left(\psi C^{*\,\epsilon}_{D;t+2;i} + (1-\psi) C^{*\,\epsilon}_{F;t+2;i}\right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{F;t+2;i}}\right]}{\sum_{i=1}^{N}\left[C^{*\,\epsilon-1}_{F;t+2;i} \left(\psi C^{*\,\epsilon}_{D;t+2;i} + (1-\psi) C^{*\,\epsilon}_{F;t+2;i}\right)^{\frac{1-\gamma}{\epsilon}-1} \frac{1}{P_{F;t+2;i}S_{t+2;i}}\right]} \\
&= \frac{\sum_{i=1}^{N}[W_{F;t+2;i}]}{\sum_{i=1}^{N}[W_{F;t+2;i}/S_{t+2;i}]}. \qquad (41)
\end{aligned}
$$

Taking into account that under rational expectations $E_t[W_{t+1}] = \Psi_1 a_t + E_{t-1}[W_{t+1}]$, where a_t is a white noise, the expression of the two period forward price in $t-1$ is:

$$
F_{t-1;2} = \frac{\sum_{i=1}^{N}[W_{D;t+1;i}] - \Psi_{D;1}\left(W_{D;t} - \sum_{i=1}^{N}[W_{D;t;i}]\right)}{\sum_{i=1}^{N}[W_{D;t+1;i}/S_{t+1;i}] - \bar{\Psi}_{D;1}\left(W_{D;t}/S_t - \sum_{i=1}^{N}[W_{D;t;i}/S_{t;i}]\right)}, \qquad (42)
$$

or equivalently for the foreign consumer:

$$
F_{t-1;2} = \frac{\sum_{i=1}^{N}[W_{F;t+1;i}] - \Psi_{F;1}\left(W_{F;t} - \sum_{i=1}^{N}[W_{F;t;i}]\right)}{\sum_{i=1}^{N}[W_{F;t+1;i}/S_{t+1;i}] - \bar{\Psi}_{F;1}\left(W_{F;t}/S_t - \sum_{i=1}^{N}[W_{F;t;i}/S_{t;i}]\right)}. \qquad (43)
$$

Next, we proceed as follows:

i)We posit initial conditions for the parameters $\{\Psi^{(0)}_{D;1}\,\bar{\Psi}^{(0)}_{D;1}\,\Psi^{(0)}_{F;1}\,\bar{\Psi}^{(0)}_{F;1}\}$.

ii) Also, we need an initial vector. Let us to denote it by $\{F_{0;1}; F_{-1;2}; T_{0;1}; T_{-1;1}\}$. Then, one hundred realizations of $C_{D;1;i}$; $C_{F;1;i}$; $C^{*}_{D;1;i}$; $C^{*}_{F;1;i}$; $Y_{1;i}$; $Y^{*}_{1;i}$ in $t = 1$ trough equations (11), (14) and the following expressions:

$$
\begin{aligned}
Y_{1;i} &= M_{1;i} + T_{0;1}\left(\frac{S_{1;i} - F_{0;1}}{F_{0;1}}\right) + T_{-1;2}\left(\frac{S_{1;i} - F_{-1;2}}{F_{-1;2}}\right); \\
Y^{*}_{1;i} &= M^{*}_{1;i} - T_{0;1}\left(\frac{S_{1;i} - F_{0;1}}{F_{0;1}S_{1;i}}\right) - T_{-1;2}\left(\frac{S_{1;i} - F_{-1;2}}{F_{-1;2}S_{1;i}}\right);
\end{aligned}
$$

$$C_{D,1,i} = \frac{\left(Y_{1,i} P_{D,1,i}^{-\sigma}\right)}{P_{D,1,i}^{1-\sigma} + \left(\frac{1-\omega}{\omega}\right)^{\sigma} (S_{1,i} P_{F,1,i})^{1-\sigma}}$$

$$C^{*}_{D,1,i} = \frac{\left(Y^{*}_{1,i} S_{1,i} P_{F,1,i}^{-\sigma}\right)}{P_{D,1,i}^{1-\sigma} + \left(\frac{1-\omega}{\omega}\right)^{\sigma} (S_{1,i} P_{F,1,i})^{1-\sigma}}$$

iii) With the previous data set, $\{C_{D,1,i}; C_{F,1,i}; C^{*}_{D,1,i}; C^{*}_{F,1,i}; Y_{1,i}; Y^{*}_{1,i}\}_{i=1}^{100}$, we iterate using the Gauss-Newton algorithm in the system concerning equations (38), (39), (42) and (43). After achieving the fixed point in the space $(F_{1,1}; F_{0,2}; T_{1,1}; T_{0,2})$ and evaluating in $t = 1$ with the variables $\{F_{1,1}; F_{0,2}; T_{1,1}; T_{0,2}\}$ the corresponding expressions, it is possible to compute values for $C_{D,1}; C_{F,1}; C^{*}_{D,1}; C^{*}_{F,1}; Y_1; Y_1^{*}$, independently of the realization values.
iv) The steps ii) and iii) are repeated recursively for each time period, allowing to obtain the numerical solutions for the remainder of the sample size, that is, $\{C_{D,t}; C_{F,t}; C^{*}_{D,t}; C^{*}_{F,t}; Y_t; Y_t^{*}\}_{t=2}^{100}$. However, this solution depends on the initial condition $\{a^{(0)}_{D,1}\ \tilde{a}^{(0)}_{D,1}\ a^{(0)}_{F,1}\ \tilde{a}^{(0)}_{F,1}\}$. To filter this effect, we estimate an autoregressive process for the expressions of $W_{D,t}; (W_{D,t}/S_t); W_{F,t}; (W_{F,t}/S_t)$ that can be computed with the simulated series of the previous solution. We use five lags in the AR specification, a robust structure in order to forecast the previous expressions. With the fitted autoregressive processes, estimation of a's are recovered to evaluate the discrepancy with $(a^{(0)}_{D,1}; \tilde{a}^{(0)}_{D,1}; a^{(0)}_{F,1}; \tilde{a}^{(0)}_{F,1})$ using the euclidean norm. The used convergence criterion is 10^{-6}. When the norm is lower, $\{C_{D,t}; C_{F,t}; C^{*}_{D,t}; C^{*}_{F,t}; Y_t; Y_t^{*}\}_{t=1}^{100}$ is the final numerical solution, whereas the norm is higher we back to step i) to iterate with the new initial condition for the vector $\{a^{(0)}_{D,1}\ \tilde{a}^{(0)}_{D,1}\ a^{(0)}_{F,1}\ \tilde{a}^{(0)}_{F,1}\}$.

Appendix 2. Statistical Tables.

Table 1. Risk premium volatility

	regression with $l=1$		regression with $l=2$	
	$\phi=0.9$	$\phi=0.1$	$\phi=0.9$	$\phi=0.1$
$\sigma^2_M = 0.005; \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_{M^*} = \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0017	0.0017	0.0012	0.0012
$\sigma^2_M = 0.005; \rho_M = 0.9; \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_{M^*} = \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0092	0.0092	0.0168	0.0169
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = 0; \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0019	0.0019	0.0023	0.0022
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = 0.9; \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0008	0.0008	0.0006	0.0006
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = -0.9; \rho_M = \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0020	0.0020	0.0020	0.0020
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = 0; \rho_M = 0.9; \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0074	0.0074	0.0153	0.0154
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = 0.9; \rho_M = 0.9; \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0094	0.0094	0.0187	0.0188
$\sigma^2_M = \sigma^2_{M^*} = 0.005; \rho_{MM^*} = -0.9; \rho_M = 0.9; \rho_{M^*} = \rho_X = \rho_{X^*} = 0; \sigma^2_X = \sigma^2_{X^*} = 0$				
$[Var(f_{t,l} - E(s_{t+l}))]^{\frac{1}{2}}$	0.0070	0.0069	0.0127	0.0109

Note: ϕ measures the degree of substitutability or complementary.

Appendix 3. Figures

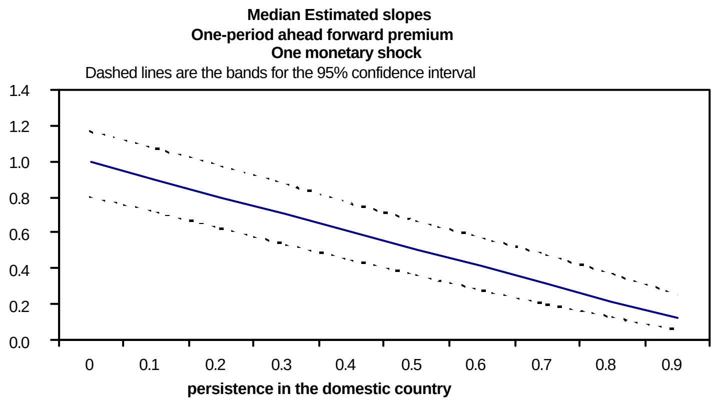

Figure 1

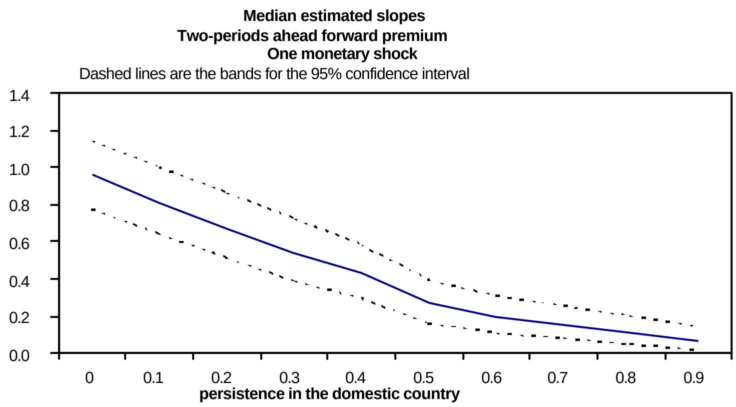

Figure 2

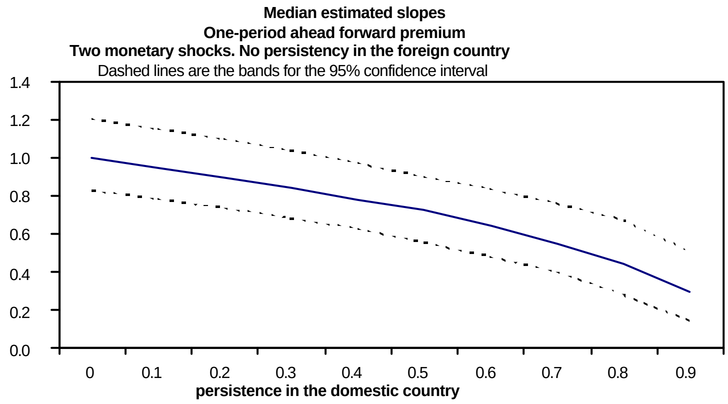

Figure 3

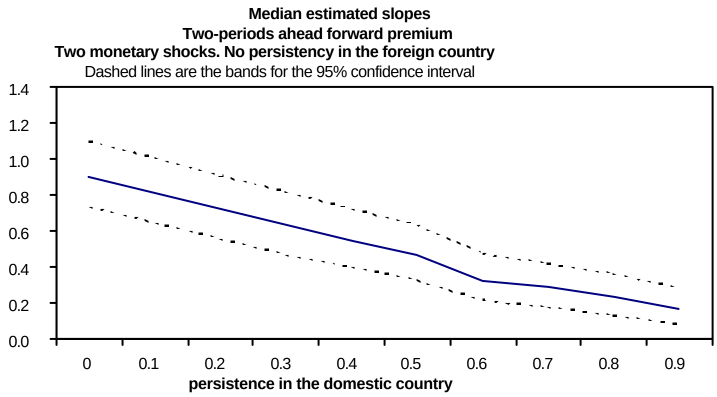

Figure 4

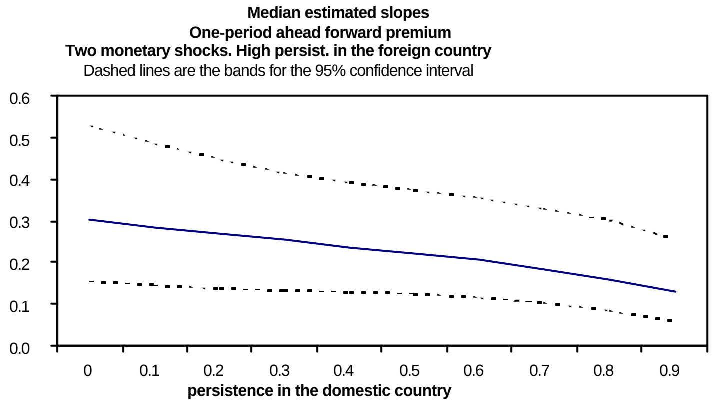

Figure 5

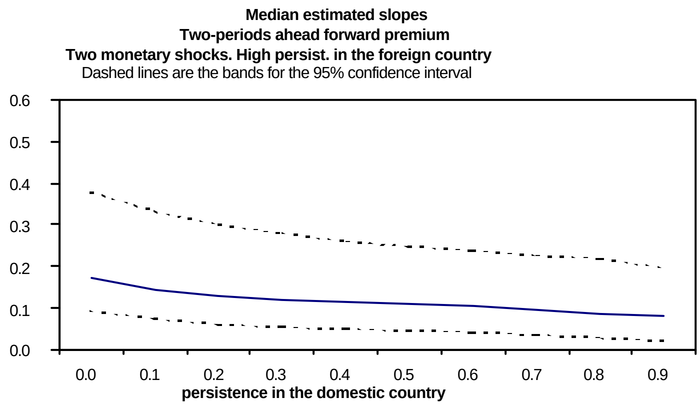

Figure 6

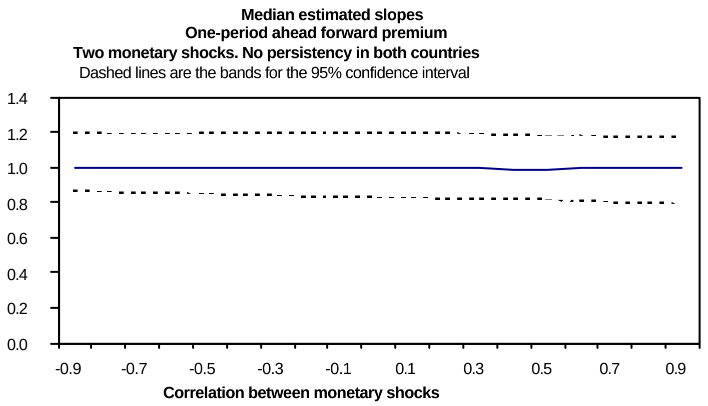

Figure 7

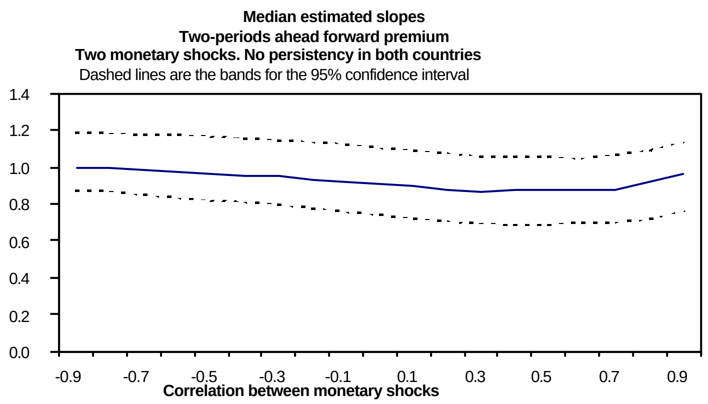

Figure 8

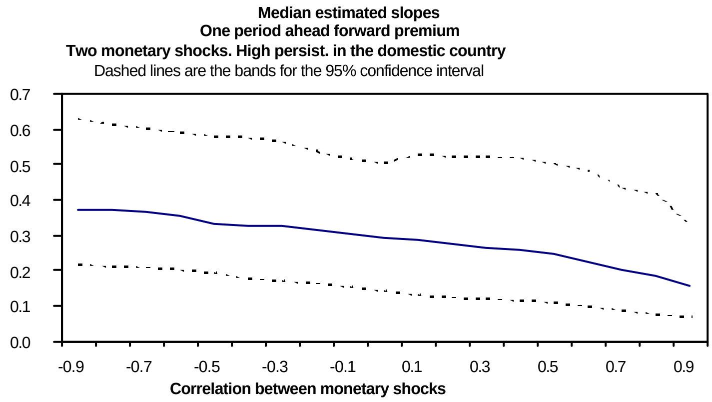

Figure 9

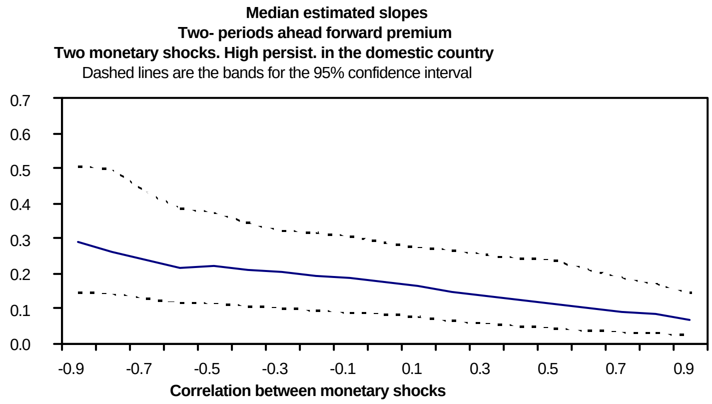

Figure 10

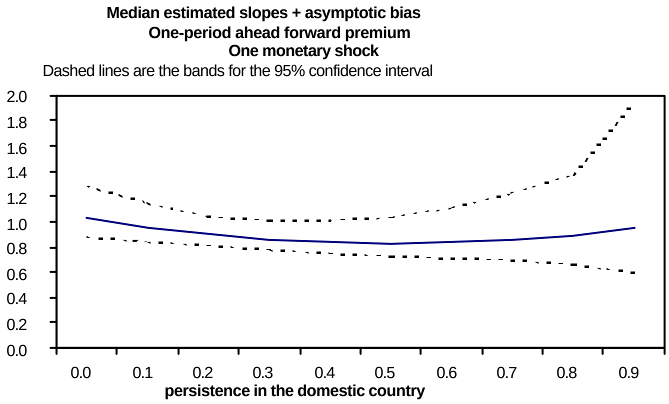

Figure 11

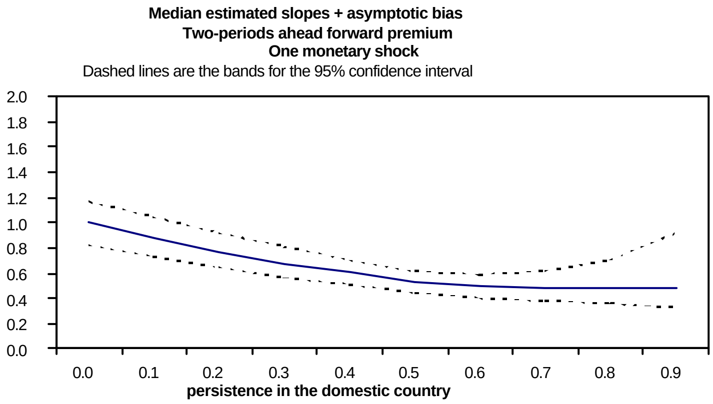

Figure 12

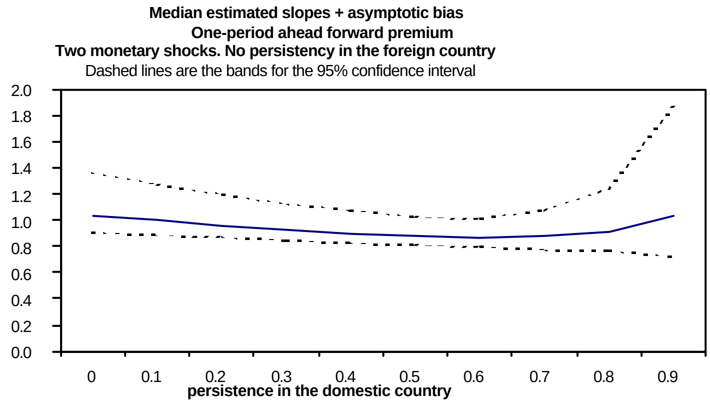

Figure 13

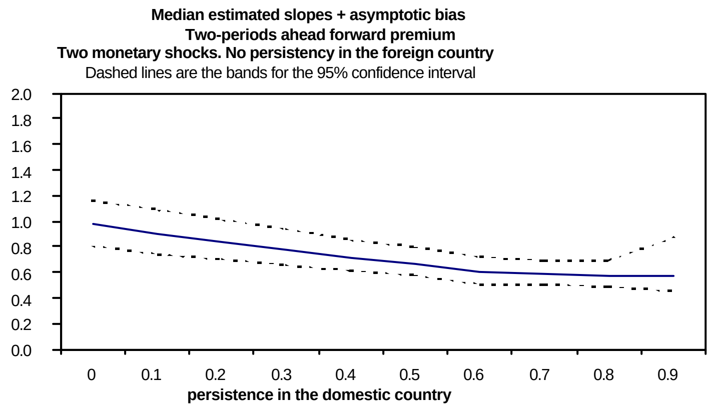

Figure 14

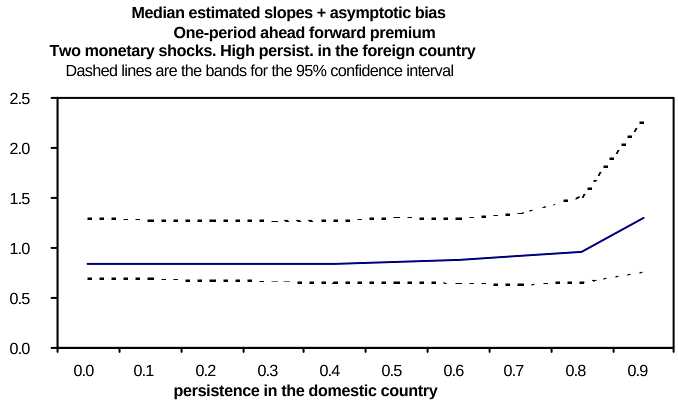

Figure 15

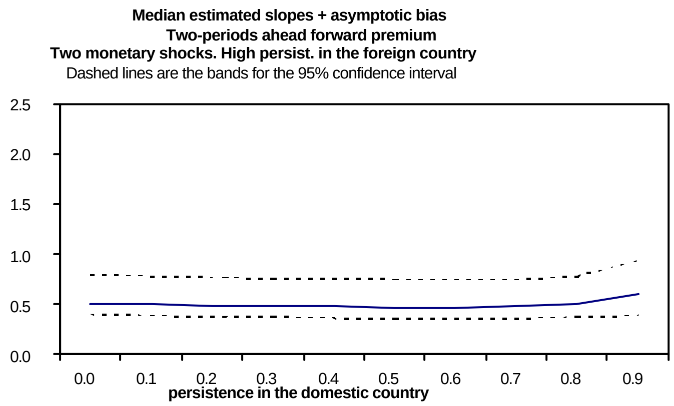

Figure 16

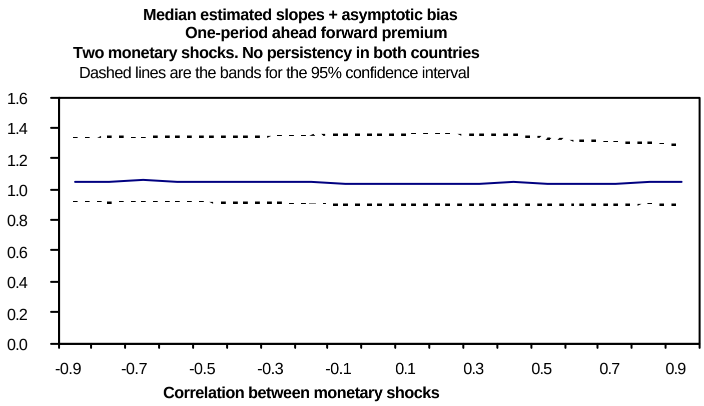

Figure 17

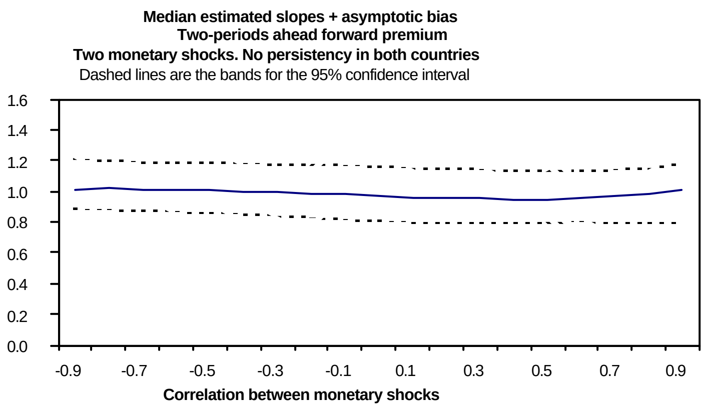

Figure 18

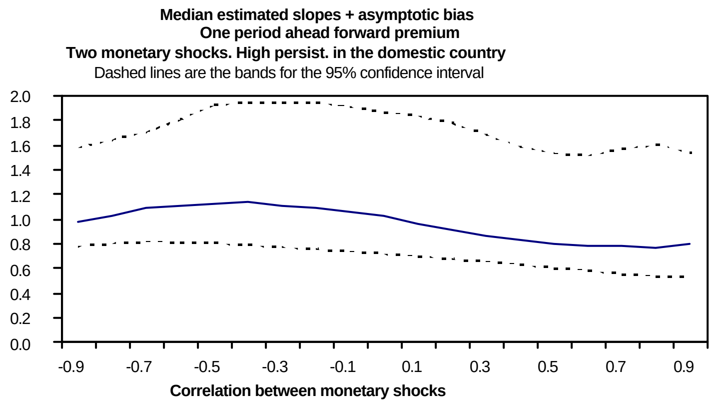

Figure 19

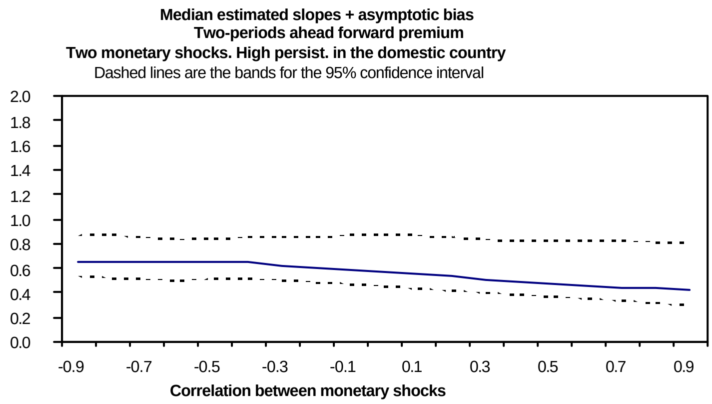

Figure 20

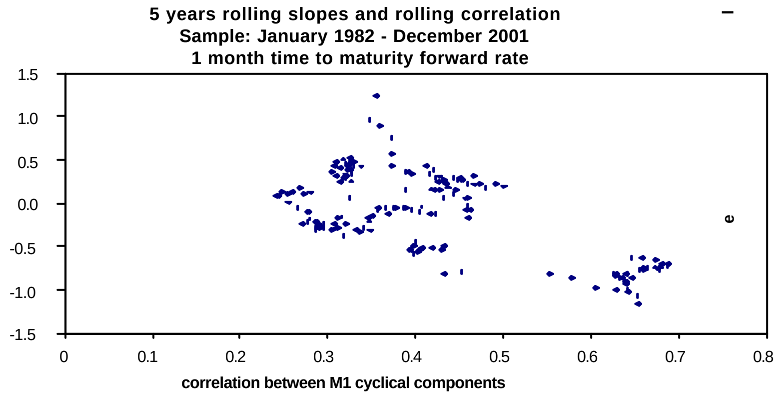

Figure 21

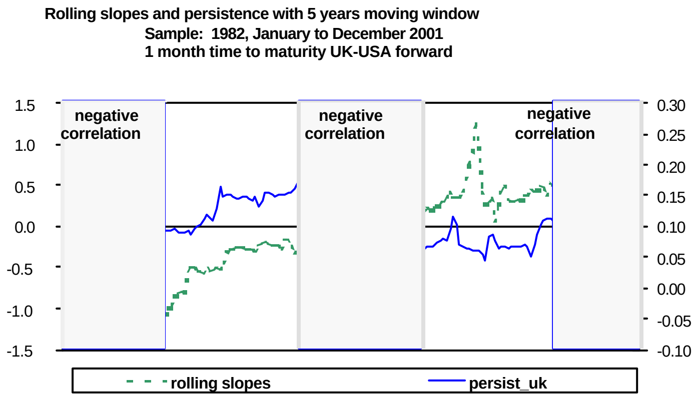

Figure 22