Non-linear adjustment to purchasing power parity: an analysis using Fourier approximations[♣]

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Abstract

This paper estimates the dynamics of adjustment to long run purchasing power parity (PPP) using data for 18 mayor bilateral US dollar exchange rates, over the post-Bretton Woods period, in a non-linear framework. We use new unit root and cointegration tests that do not assume a specific non-linear adjustment process. Using a first-order Fourier approximation, we find evidence of non-linear mean reversion in deviations from both absolute and relative PPP. This first-order Fourier approximation allows us to capture many features of the non-linear decay detected in the data. Our results are consistent with theoretical arguments on international goods markets arbitrage under transaction costs as well as with an emerging strand of empirical literature. In this sense, this paper contributes towards forming a consensus on the presence of nonzero transaction costs across a broad range of countries.

Keywords: Unit-root test, Cointegration test, Fourier approximation, nonlinear model, exchange rates, purchasing power parity.

JEL classification: F31, C22, C32

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1. INTRODUCTION

There is a large and growing literature indicating that traditional linear unit roots tests do not work well when there are important non-linearities in the data. This problem appears frequently in the analysis of the theory of Purchasing Power Parity (PPP), which states that the real exchange rate should equal one, or at least have a tendency to return quickly to one when that long-run ratio is disturbed for some reason. Sometimes this version of PPP is called absolute PPP. Relative PPP is the weaker statement that changes in national price levels are always equal or at least, tend to equality over sufficiently long periods.

The rationality behind the PPP is a simple arbitrage hypothesis: If two identical goods are traded at different prices in different countries, a profitable arbitrage opportunity arises and the arbitrageurs can buy the good cheaply in one location and sell it at a higher price in the other. In the absence of arbitrage costs, this process leads to convergence of the deviations from PPP towards zero. Exchange-rate-adjusted prices are equalized across countries, leaving no room for profitable arbitrage opportunities. Under this version of the PPP theory, one would expect stationarity in real exchange rate dynamics.

The parity condition rests on the assumption of perfect inter-country commodity arbitrage and it is a central building block of many theoretical and empirical models of exchange rate determination. Due to factors like transaction costs, taxation, subsidies, actual or threatened trade restrictions, the existence of non traded goods, imperfect competition, foreign exchange market intervention, and the different composition of market baskets and price indices across countries, one may expect PPP to be valid only in the long run. Then, prices and nominal exchange rate should show a steady long-term relationship, and therefore one would expect cointegration between them.

Empirical studies over long periods of time have supported long-run PPP (Diebol *et al.,* 1991, Taylor 1996, Michael *et al.*, 1997). However, results are mixed when the recent floating-rate period is examined. Using standard unit roots tests Corbae and Ouliaris (1988), Meese and Rogoff (1988), Edison and Fisher (1991) and Grilli and Kaminsky (1991) cannot reject the unit root null hypothesis for real exchange rates in the managed-float regime. In contrast, Perdroni (1997), Frankel and Rose (1996), Lothian (1997), Oh (1996), Wu (1996) and Papell and Theodoridis (1998) find strong evidence of mean reversion in real exchange rates by implementing panel data variants of standard unit-root tests. However, O’Connel (1998) strongly disputes these mean-reversion findings in real exchange rates as they fail to control for cross-sectional dependence in the data. Additional evidence against reversion to

PPP based on a panel has been reported in Engel *et al.* (1997). Papell (1997) and Liu and Maddala (1996) also find that evidence of mean reversion in panels of real exchange rates is very sensitive to the group of countries considered.

Recently, an alternative explanation bases the persistent deviations from parity on the presence of market frictions that preclude commodity trade. Dumas (1992), Uppal (1993), Sercu *et al.* (1995) develop equilibrium models of real exchange rate determination which take into account transaction costs and show that adjustment of real exchange rate towards PPP is necessarily a nonlinear process. Market frictions in international trade introduce a neutral range, or band of inaction, within which deviations from PPP are left uncorrected, as they are not large enough to cover transaction costs. In this dynamic equilibrium framework, deviations from PPP follow a nonlinear stochastic process that is mean reverting.

Obstfeld and Taylor (1997), and O'Connell and Wei (2002) provide some empirical evidence of the effect of transaction costs in this context. In all these studies, the nonlinear nature of the adjustment process is investigated in terms of a TAR model (Tong, 1990). The TAR model allows for a transaction costs band within which no adjustment in deviations from the PPP takes place –so that deviations may exhibit a unit root behavior– while outside the band, as goods arbitrage becomes profitable, the process switches abruptly to become stationary. Other authors, as Michael *et al.* (1997), Baum *et al.* (2001), employ the exponential smooth transition autoregression (ESTAR) framework to analyze the dynamic behavior of deviations form PPP, finding evidence of nonlinear adjustment.

Overall, one way to circumvent the assumption of a linear dynamic process is to posit a particular model of non linear adjustment. However, if there is little a priori information concerning the actual form of adjustment, the estimated model is likely to suffer from a misspecification error. Moreover, it is often difficult to discriminate among alternative non-linear models using standard diagnostic tools.

In this paper, we test non-linear price adjustment mechanisms under the current float, employing first order Fourier series to approximate the non-linear adjustment process. Under this specification, it is not necessary to pre-specify the nature of the adjustment process. This feature makes this procedure more suitable than other alternatives such as TAR or ESTAR. As Enders and Ludlow (2002) remarks, if there is reversion — but not linear or threshold reversion—the Fourier approximation might best characterize the adjustment process, since the functional form of the alternative hypothesis need not be specified.

The paper is organized as follows: Section 2 presents a brief description of absolute and relative PPP. Section 3 analyzes the data testing PPP using linear tests. Section 4 describes a procedure that can capture the presence of non-linear adjustment and presents the main results. Finally, Section 5 concludes.

2. BRIEF DESCRIPTION OF PPP

The key theoretical concept underlying our analysis is the Purchasing Power Parity. In its simplest formulation, PPP posits equality between the price level in one country and the exchange rate adjusted price level in the other. It therefore treats the real exchange rate — the nominal exchange rate divided by the ratio of the two countries price levels — as a constant.

How close this description is to actual experience depends on how the real exchange rate behaves. A common strategy to test for PPP hypothesis over the past decade and a half consists on analyzing the presence of a unit root in the real exchange rate. To understand the relationship between real exchange rates and PPP, consider the following expression for the real exchange rate:

$$q_t = s_t - p_t + p_t^*, \qquad (1)$$

where q is the log of the real exchange rate, s is the log of the corresponding nominal exchange rate measured in units of domestic currency per unit of foreign currency, and p^* and p, are the logarithms of foreign and domestic prices, respectively. The absolute PPP hypothesis states that q will equal a constant, call $\overline{q}$.[1] In this case, we can rewrite (1) as:

$$s_t = \overline{q} + p_t - p_t^*, \qquad (2)$$

Under floating exchange rates, (2) is a relationship among price levels and the nominal exchange rate. We can thus think of the above equation as defining a cointegration relationship between these three variables. To see this more clearly, consider the following stochastic equation analogue to (2):

[1] Strictly speaking, the absolute PPP hypothesis states that the exchange rate between the currencies of two countries should equal the ratio of the price levels of the two countries, specifically: $S_t = P_t / P_t^*$, where S, is the nominal exchange rate measured in units of domestic currency per unit of foreign currency, P is the price level in the domestic country, and P^* is the price level in the foreign country. The measures of consumer prices published by national statistical agencies are typically reported as indexes relative to a base year (say, 1995=100). Thus, they only measure the rate of change of the price level from the base year, not its absolute level. For this reason, it is difficult to test absolute PPP in a strict sense.

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + e_t, \qquad (3)$$

where β_i are coefficients and e_t is an error term. If we impose the condition $\beta_1 = 1$ y $\beta_2 = -1$, it is possible to apply a simple unit root test to the real exchange rate itself to test for absolute PPP.

That hypothesis is very restrictive and recent studies on the validity of PPP have focused in a weaker theory: the relative PPP hypothesis, which states a long-run relationship between nominal exchange rate and prices. Relative PPP hypothesis requires that e_t in (3) be a stationary process, that is, (s_t, p_t, p_t^*) should form a cointegrated system (Engle and Granger, 1987).

3. DATA AND PRELIMINAR PPP ANALYSIS

In this paper, both the relative and the absolute PPP hypothesis are tested using the Consumer Prices Index (CPI) as a proxy for price levels of each country's output. The nominal exchange rates are end-of-month bilateral US dollar exchange rates. In all cases, the USA is considered the foreign country.

EcoWin provides us with all series extracted from the International Monetary Fund's *International Financial Statistics* database. The sample included data for 19 industrialized countries: the USA, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Mexico, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The database spans the period January 1973 to February 2005 for Canada, Denmark, Japan, Norway, Sweden, Switzerland and the United Kingdom. For Mexico, the sample starts in January 1981 and finishes in February 2005, for Greece, it starts in April 1981 and finishes in December 2001, and for the Netherlands the sample spans from January 1982 to December 1998.[2] For Austria, Belgium, Finland, France, Germany[3], Italy, Portugal, and Spain the sample spans from January 1973 to December 1998 (the Euro currency is

[2] The Netherlands fit their currency to the German Mark up to January 1981. April 1981 is the date of the entry of Greece into the European Economic Community. After the country's public finances went bankrupt in 1982, Mexico began a profound transformation. The foreign debt crisis of 1981-82 had a severe and irreversible effect on public finances and the State's economic and political thinking. Until that date, Mexico's economy and politics were heavily subsidized and protected from competition.

[3] CPI data for the unified Germany are only available from January 1991.

created in January 1999). Figure 1 shows the standardized real exchange rates for the eighteen countries. The most relevant point here is that each real exchange rate has exhibited large and persistent deviations around its mean. Nevertheless, during this period the real exchange rates considered seem to revert to an unknown reference value.

[Insert Figure 1]

We examine the evidence on the absolute PPP hypothesis testing for stationarity of the real exchange rates using the Augmented Dickey Fuller (ADF) and the more efficient Dickey Fuller Generalized Least Squares (DF-GLS), proposed by Elliott, Rothenberg and Stock (ERS, 1996).[4,5] Table 1 presents both tests with and without a time trend in the test regression.[6] The lag length for the differenced dependent variable on the right hand side of the test regression is determined using the Akaike information criterion (AIC) in both tests. The tests produce little support for absolute PPP version. ADF test never rejects the null of non-stationarity, while the DF-GLS only does it for four countries. For Germany and Norway the DF-GLS test rejects the null hypothesis of a unit root at the 5% significance level. For Greece and Italy, this test rejects the null hypothesis at the 10% significance level.

Cheung *et al.* (2003) test for a unit root on the real exchange rate for five countries: France, Germany, Italy, Japan, and the United Kingdom. They find stationarity on real exchange rate for all cases, except Japan. For France, Germany and Italy their sample spans the same period of time than ours. In contrast to Cheung *et al.* (2003), we do not find stationarity of the real exchange rate for France (in this case we detect a different lag in the DF-GLS test).

[Insert Table 1]

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[4] DF-GLS test has substantially improved power when an unknown mean or trend is present. As Elliot *et al.* (1996) proves, the modified test works well in small samples.

[5] All individual series (nominal exchange rates and prices) have also been tested for the presence of unit root using the ADF and DF-GLS tests. Consistent with the literature, the null hypothesis of a single unit root cannot be rejected in any case. To save space, these results are not reported here, but are available upon request.

[6] Some researchers, for example, Cheung and Lai (1998), and Koedijk *et al.* (1998), have found that the stochastic processes of some of the real exchange rates cannot be adequately modeled without the inclusion of a linear deterministic time trend. The linear deterministic time trend is generally interpreted as representing systematic differences in productivity growth between tradable and non-tradable goods in the two countries. On the other hand, other researchers, for example, Papell and Theodoridis (1998), and Amara and Papell (2002) consider a linear time trend in the real exchange rate as inconsistent with long-run PPP.

We also test relative PPP using Engle and Granger (1987) methodology. Firstly, we estimate expression (3) and then we test for stationarity of the residual using the ADF statistic. Figure 2 shows the estimated standardized disequilibrium series for all countries.

[Insert Figure 2]

Additionally, we use the cointegration DF-GLS test developed by Perron and Rodríguez (2001). They propose a residual based test for cointegration when residuals are constructed using GLS detrended o quasi-differenced data to each variable of the system.[7] As Table 2 reports, the Engle and Granger (1987) test rejects the null of non-cointegration only in one case, the Netherlands, and the DF-GLS in merely two cases (Denmark and the Netherlands). These results fall short of being supportive of long-run PPP.

[Insert Table 2]

In summary, the evidence in favor of mean reversion in US dollar-based PPP deviations series is quite weak (we reject the absolute PPP except for 4 countries and only for 2 cases we can not reject the relative PPP) for the sample period studied here.

As mentioned in previous sections, several recent studies propose an alternative explanation that bases the persistence of managed-float deviations from parity on the presence of market frictions that impede commodity trade. Several models take into account transaction costs and show that adjustment to the equilibrium is necessarily a non-linear stochastic process. However, this non-linear, mean-reverting, process is hard to capture using linear unit root tests.

4. NON-LINEAR ADJUSTMENT

To test for stationarity of the deviations from PPP, we apply the Enders and Ludlow (2002) test (EL-test). This procedure allows us to study the non-linear mean-reverting behavior of PPP without having to specify the kind of nonlinear adjustment process. Enders and Ludlow (2002) suggest the following modification of the Augmented Dickey-Fuller (ADF) test:

[7] Perron and Rodríguez (2001) analyze residual based tests for cointegration. Among other cases, they consider the standard ADF and derive their asymptotic distribution assuming a general quasi-differencing parameter $\bar{c}$ and tabulate its critical values. Their simulations reveal an important power gain from using GLS detrended data, especially if the quasi-difference parameter is set as suggested by Elliot *et al.* (1996).

$$y_t = \alpha(t)\ y_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta y_{t-i} + \varepsilon_t, \tag{4}$$

where $\{y_t\}$ is the process of interest (the real exchange rate in our case), and ε_t is the stochastic error term. No specification of the functional form of *α(t)* is required, since this can be approximated by a sufficiently long Fourier series. However, in order to keep the problem tractable, we consider only a Fourier approximation using a single frequency as Enders and Ludlow (2002) suggest:

$$\alpha(t) = a_0 + a_1 \ \text{sen} \frac{2\ \pi\ k}{T} t + b_1 \ \cos \frac{2\ \pi\ k}{T} t, \tag{5}$$

where *k* is an integer number in the interval [1, *T*/2]. If y_t denotes the real exchange rate, we can use (4) to test the absolute PPP hypothesis.

To analyze the relative PPP hypothesis, the EL-test can be easily generalized to test for cointegration in the Engle and Granger (1987) framework. Let the $\{\hat{e}_t\}$ sequence denote deviations from long-run equilibrium. The relationship among exchange rates and prices are estimated by OLS in model (3) as:

$$\hat{e}_t = s_t - (\hat{\beta}_0 + \hat{\beta}_1 p_t + \hat{\beta}_2 p_t^*), \tag{6}$$

where s_t , p_t and p^*_t have been defined above. The non-linear adjustment process is:

$$\hat{e}_t = \alpha(t)\, \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta \hat{e}_{t-i} + \varepsilon_t, \tag{7}$$

where α(t) is defined in (5).

Instead of searching for a specific nonlinear adjustment, the problem is reduced to finding the most suitable values of a_0, a_1, *b* and *k* using equations (4) [or (7)] and (5).[8] The sufficient and necessary condition for a non-explosive adjustment process, or, in other words, for a mean-reverting behavior is:

$$\begin{aligned} &|a_0| < 1 + r^2/4 \ \text{ for } \ r \leq 2 \\ &r = \sqrt{a_1^2 + b_1^2}. \end{aligned} \tag{8}$$

An important feature of the EL-test is that it allows for different patterns of mean-reverting behavior. In particular, the test allows for nonlinear autoregressive decay with

[8] The ADF test is obtained for the special case of $a_1 = b_1 = 0$.

possible periods of explosive and/or oscillatory behavior. In particular, for $a_0 > 0$ and $r < 2$ the series reverts to an attractor if $a_0 < 1 + r^2/4$. There are two main different types of decay:

1. If $a_0 + r < 1$ there is overall reversion that can be monotone when $a_0 > r$ and oscillatory when $a_0 < r$.
2. If $a_0 + r > 1$ there will be k periods when $\alpha(t)$ exceeds unity. This implies that while the overall process ultimately reverts to the attractor (directly when $a_0 > r$ and with oscillations when $a_0 < r$), the sequence exhibits periods of explosive behavior.[9]

This feature makes this test especially relevant in the analysis of the theory of long-run PPP. As we remarked above, international trade is affected by factors like transaction costs, taxation, subsidies, trade restrictions or official intervention in the exchange markets that could cause asymmetric movements in exchange rates. These effects make deviations from PPP a mean-reverting non-linear stochastic process, because exchange rates and domestic and foreign prices adjust to past disequilibria as a function of multiple and complex factors. Sometimes, the real exchange rate can be stationary but frictions in international trade introduce some periods of inaction, within which the mispricing is left uncorrected.

Also, the properties of the test make this approach more suitable than other alternatives such as TAR or ESTAR. With this procedure, we do not need to specify the functional form of the alternative hypothesis. As Enders and Ludlow (2002) remark, if there is mean reversion but the decay is not linear or it does not presents threshold reversion then the Fourier approximation might best characterize the adjustment process.

We apply the EL-test to both absolute and relative PPP. For absolute PPP, we run a regression of the real exchange rate over a constant and a linear trend and compute the $\hat{e}_t$ series.[10] Then, the following regression is estimated for all the integer values of k in the interval 1 to $T/2$, to select the most suitable k frequency:

$$\Delta \hat{e}_t = \left[c + a_1 \operatorname{sen} \frac{2\pi k}{T} t + b_1 \cos \frac{2\pi k}{T} t \right] \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta \hat{e}_{t-i} + \varepsilon_t , \qquad (9)$$

[9] Note that (5) has the equivalent representation: $\alpha(t) = a_0 + r\cos\left(\frac{2\pi kt}{T} + d\right)$, where $r = \sqrt{a_1^2 + b_1^2}$ and $d = \arcsin(a_1 / r)$. If $a_0 + r > 1$ and since $\cos\left(\frac{2\pi kt}{T} + d\right)$ can equal unity, there will k periods when $\alpha(t)$ exceed the unity, there being explosive periods. Oscillations appear when $\alpha(t) < 0$, that being the case when $a_0 < r$ and $\cos\left(\frac{2\pi kt}{T} + d\right) = -1$.

[10] Additionally, we apply the test on the demeaned real exchange rate. To save space, we only present the demeaned and detrended case, but results are available upon request.

where $c = a_0 - 1$ and p-1 is the number of lags needed to completely eliminate residual autocorrelation.[11] In the case of relative PPP, $\hat{e}_t$ denote the residuals of the regression of the deviations from PPP defined in (6) on a linear trend.[12] We choose the value of k that minimizes the sum of the squared residuals. This value is denoted k^*, and the coefficients linked to such frequency by c^*, a_1^* and b_1^*. The period, T/k^*, indicates the length of time required for the process to repeat a full cycle. Therefore, the bigger is k^*, the smaller is the period.

In order to test for stationarity of deviations of PPP, we compute three statistics from the estimation of (9): *F_all, F_trig,* and *τ-statistic.*[13] The test procedure has the following structure. First, we compute the *F_all* statistics, this is an *F* statistic for the null hypothesis $c^* = a_1^* = b_1^* = 0$. This statistic is used to test whether the series in question is a random walk. If the *F_all* null hypothesis is rejected, the series in question exhibits mean-reversion, which may be, linear, non-linear or both. Second, if the *F_all* null hypothesis is not rejected then we compute the *F_trig* statistic for the null hypothesis $a_1^* = b_1^* = 0$, which imply $r = 0$. If this hypothesis is rejected, there is a non-linear mean-reverting behaviour in the data. Finally, if the *F_trig* null hypothesis is not rejected then we compute the *τ-statistic* for the null hypothesis $c^* = 0$. Rejecting the hypothesis $c^* = 0$ is a necessary condition for mean-reversion only when $r = 0$.[14] If this null hypothesis is also not rejected then we rejects the PPP hypothesis.

The power of the three tests depends on the values of r and c^*. As r increases, the power of *F_all*, *F_trig* and *τ-statistic* increases. Besides, for small values of c^* and r, the power of *F_trig* is slightly higher than *F-all* and the *τ-statistic.*

Additionally, when the series revert to the mean, we discuss the different types of decays: monotone decay, decay with explosive periods, oscillations, and oscillations an explosive periods. The type of reversion depend on the values of c^* and r.

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[11] Estimation is carried out in differenced data to allow for comparison with the Engle and Granger (1987) test for the linear case.

[12] We also analyze the demeaned relative PPP. The results are almost the same that in the detrended case. For this reason, to conserve space, these results are not reported here, but are available upon request.

[13] Enders and Ludlow (2002) obtain the critical values for these three statistics.

[14] However, if the *F_trig* test correctly indicates $r > 0$, rejecting the null of the c^* test in favor of the alternative hypothesis $c < 0$, is sufficient to guarantee reversion.

4.1. Results for absolute PPP

EL-test results for absolute PPP are shown in Table 3. They provide stronger evidence on mean reversion in real exchange rates than linear unit root test. In particular, *F_all* rejects the null of a nonstationary behavior for Austria, Belgium and Switzerland. Additionally, the *F_trig* detects nonlinear adjustment processes for seven countries (Belgium, Canada, Denmark, France, Norway, Portugal and Switzerland). Also, the values of the statistic are very high for Austria and Japan indicating evidence on non-linearity.

[Insert Table 3]

As Table 3 shows, it is not possible to reject the null $c^*= 0$ in any case. This statistic has a low power when a^*_0 is near 0.9, as is our case. This finding is consistent with the results of the linear unit-root tests shown in Table 1.

In the other hand, the value of a_0^* fulfils the reversion condition in (8) for all the countries. Nevertheless, in the case of Italy, Mexico, Portugal, Spain and Sweden the value of a_0^* is over 0.99. The estimated values of k^* oscillate between 3 and 173. The minimum value is achieved for Canada, Spain and the United Kingdom, and the maximum for Norway.

In terms of the pattern of decay detected, in all cases $a^*_0>r$ and $a^*_0+r >1$, indicating the existence of periods of explosive behavior in real exchange rates. These finding indicate the presence of periods of decay and periods of non-stationarity, this being compatible with the existence of market frictions and asymmetries that make arbitrage profitable only under some conditions.

Summarizing, using a nonlinear analysis that extends the traditional ADF test, we find stronger evidence in favor of absolute PPP than in the linear one. We find evidence in favor of mean-reversion in seven real exchange rates whereas the standard ADF test does not reject the non-stationarity null in any country. In the linear case, we find some evidence in favor of absolute PPP using DF-GLS test for Germany, Greece, Italy and Norway whereas with the EL-test we find that evidence for Austria, Belgium, Canada, Denmark, France, Norway, Portugal and Switzerland.

4.1. Results for relative PPP

When we analyze the deviations from PPP in the cointegration framework, the evidence on non-linear mean reverting is higher. As can be seen in Table 4, the *F_all* and *F_trig* statistics provides stronger evidence on mean reversion in the deviations from relative

PPP than in the linear cointegration framework. In particular, *F_all* rejects the null of a nonstationary behavior for Greece, Mexico, the Netherlands and Switzerland at 5%. Also, the values of this statistic are close to the 10% critical value for Norway and the United Kingdom. The *F_trig* detects nonlinear decay for six countries (Greece, Italy, Mexico, Norway Switzerland and the United Kingdom). Finally, the null $c^*= 0$ is rejected at the 5% significance level for Germany, Greece and the Netherlands.

[Insert Table 4]

The estimated values of a_0^* fulfill the cointegration condition for all the countries, with lower values than in the absolute PPP case. Canada achieves the higher value (a_0^*=0.9846), and Germany the lower one (a_0^*=0.7765).

For all countries, except for Germany, $a_0 > r$ and $a_0+r > 1$. This result indicates that the deviations from PPP ultimately decay to an attractor but the sequence exhibits periods of explosive performance. In the German case, there is a monotone adjustment process.

Also in this case, using the nonlinear cointegration analysis we find stronger evidence in favor of relative PPP than under the linear framework. We find evidence in favor of mean-reversion in long-run relationship residuals for eight countries, whereas the Engle and Granger procedure only reject the null of non-cointegration for the Netherlands and Denmark.

4.3. A non-linear error correction model

Summarizing, we find evidence of nonlinear mean reverting behavior in deviations from PPP (absolute, relative or both) in twelve of the eighteen analyzed countries. These results contrast with those we found previously in the linear analysis, with evidence on a linear convergence to long run equilibrium in six countries. As expected, for those six countries we also find stationarity in deviations from PPP with the Fourier approximation, but with a non-linear component.

These results suggest stronger evidence in favor of the long run PPP than that found by another authors. For example, Baum *et al.* (2001) analyze the same panel of countries that we do, with the exception of Mexico, using a shorter sample. They find nonlinearities only in seven countries within an ESTAR framework. In our case, we find a nonlinear adjustment to PPP for the same countries, except Japan and Finland. Additionally, we find nonlinear adjustments in other seven countries.

Since in the majority of cases the analyzed exchange rates and prices are cointegrated and the adjustment towards the long-term relationship is non-linear, we can estimate an error correction model exhibiting Fourier-decay. This model allows us to distinguish between short and long term Granger causality. Long-term causality (Granger, 1986) is the result of including all variables lagged one period in the ECM. This causality will always occur at least in one direction since, according to Engle-Granger representation theorem, if two variables are cointegrated, at least one of them must respond to deviations from the long run equilibrium relationship. The specified VAR-ECM exhibiting Fourier-decay is in our case:

$$\begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} 1 \\ \text{sen}\,\frac{2\pi k^* t}{T} \\ \cos\frac{2\pi k^* t}{T} \end{pmatrix} \hat{e}_{t-1} + \sum_{i=1}^{p-1} \Phi_i \begin{pmatrix} \Delta s_{t-1} \\ \Delta p_{t-1} \\ \Delta p_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \quad (10)$$

where Φ_i, $i = 1,\ldots, p-1$ are *3x3* coefficient matrices. This model allows us to split the adjustment process to past disequilibria in three different terms: a linear one, captured by the α_{i1}, i = 1,2,3 coefficients, and two non-linear terms, captured by $\alpha_{i2}\,\text{sen}\,\frac{2\pi k^* t}{T}$ and $\alpha_{i2}\cos\frac{2\pi k^* t}{T}$, i = 1,2,3.

[Insert table 5]

In Table 5 we present estimation results for non-linear error correction model (equation (10)) for the eighteen countries analyzed. The first three columns show the estimated coefficients of the long-run relationship in expression (3). The remaining columns show estimated adjustment coefficients, which represent the adjustment to past disequilibria in the exchange rate equation (cols 5 to 7), in the domestic price equation (cols 8 to 10) and in the foreign price equation (cols 11 to 13).

The exchange rate responds to past disequilibria in all countries, suggesting that the exchange rates adjust for deviations from relative PPP. In most cases, the response of exchange rates is made up of a linear component and a non-linear one, since not only α_{11} but also α_{12} or $\alpha_{13,}$ are individually significant.[15] The evidence on price adjustment to deviations from PPP is much weaker.

Finally, analyzing results for domestic price equations we only find significant response to deviations from relative PPP for six countries: a linear response in France, Greece, Austria, and nonlinear responses in Austria, Mexico, the Netherlands and Norway.

[15] We approximate the distribution of these signification tests by a normal standard distribution.

The response of the foreign price is linear for Belgium, Denmark, Germany, Mexico and Spain, and nonlinear for France and the Netherlands.

These results suggest that exchange rates take most of the burden of adjustments to long-run PPP, in most cases through a non-linear correction mechanism. Most likely, this reflects some degree of stickiness in consumer prices in all countries.

We also analyze linear Granger causality (or short term linear causality) in order to study the temporal flow of information in the system. Using the standard definition, a variable is said to cause another, if the introduction of the lags of the causal variable in the model of the caused one improves the forecast of the caused variable.[16] Results are shown in Table 6.

[Insert table 6]

We find little evidence of bi-directional linear causality between exchange rates and prices. Lagged domestic prices changes are only significant in the equation of exchange rates for France, Germany, Portugal and Sweden. Lagged foreign prices changes are only significant for France, Japan, Norway and Switzerland. On the other hand, lagged exchange rate changes are significant in the case of Italy, Mexico, the Netherlands, Norway and Switzerland in the equation of domestic prices and only for Belgium, Denmark and Portugal in the case of foreign prices. No significant pattern seems to emerge from these observations.

Remarkably, we find short-term bilateral feedback between domestic and foreign prices changes, the lags of each one of them being jointly significant in the equation of the other for almost all the countries in the sample. This result may reflect the presence of exogenous shocks such as shocks in raw materials prices like petroleum, simultaneously affecting domestic and foreign prices.

5. CONCLUSIONS

Using nonlinear tests, this paper finds evidence of nonlinear mean-reverting behavior in real exchange rates and in deviations from long run PPP, which are consistent with the purchasing power parity hypothesis, adjusted for market frictions such as transaction costs. We use a sample of monthly CPI indexes and nominal exchange rates for 1973-2004, for a broad set of US trading partners. We find that deviations from PPP may go trough short

[16] To test the existence of short-term causality, we start from the estimated VAR-ECM model (expression (10)) and carry out a joint significance test of the lags of the causal variable in the equation of the caused one.

periods of explosive behavior, with overall mean reverting performance. These results reinforce the insight of previous studies regarding the possible presence of non-linear, but stationary adjustment processes to long-run PPP.

We find evidence of a nonlinear mechanism to correct for deviations from long-run PPP. Our findings suggest that the adjustment to long-run equilibrium comes mainly from the exchange rate market. In the short term, we find a bi-directional flow of information between domestic and foreign prices, possibly due to experiencing common shocks, while we rarely find Granger causality between price changes and nominal exchange rate changes.

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Figure 1. Standardized real exchange rates

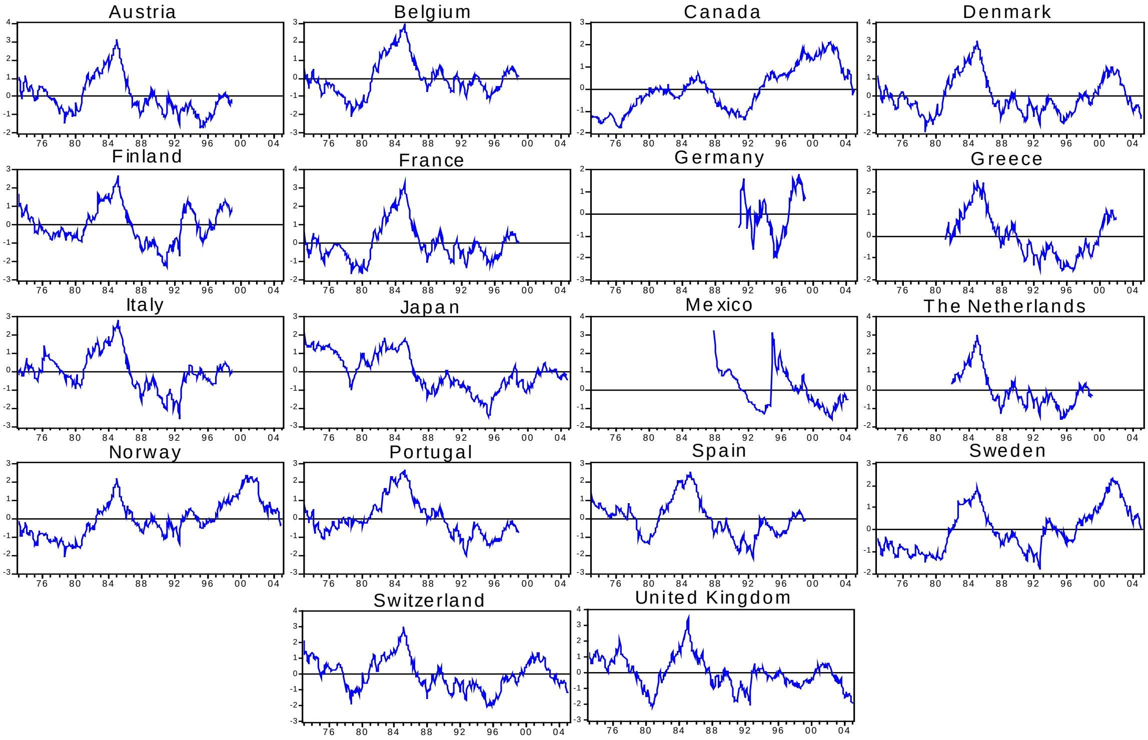

Table 1. Unit root test for the real exchange rate

	Obs	ADF_C	Lags	ADF_C+T	Lags	GLS_C	Lags	GLS_C+T	Lags
Austria	312	-1.9079	0	-1.9021	0	-1.3150	0	-1.8693	0
Belgium	312	-1.6858	0	-1.7303	0	-1.4671	0	-1.5658	0
Canada	383	-1.5311	0	-2.0496	11	-0.5701	0	-2.2358	11
Denmark	384	-1.9444	0	-1.9419	0	-0.9980	0	-1.7221	0
Finland	312	-2.2032	0	-2.1598	0	-1.0524	0	-1.4038	0
France	312	-1.9728	0	-1.9698	0	-1.6875	0	-1.8512	0
Germany	96	-2.1239	9	-2.4275	9	-2.0806**	9	-2.4260	9
Greece	249	-1.8459	14	-2.3900	13	-1.8406*	14	-1.8627	14
Italy	312	-1.8563	0	-1.9488	0	-1.8542*	0	-1.9048	0
Japan	383	-1.9905	1	-2.0807	1	-0.3946	1	-1.6933	1
Mexico	195	-2.0642	6	-3.0265	1	-0.1428	6	-1.2894	6
The Netherlands	204	-1.5446	0	-1.9558	0	-1.4286	0	-1.8244	0
Norway	384	-2.2005	0	-2.2548	0	-2.1219**	0	-2.1546	0
Portugal	312	-1.5623	0	-1.6157	0	-1.0638	0	-1.6017	0
Spain	312	-1.8427	0	-1.6791	0	-0.7866	0	-1.4246	0
Sweden	383	-1.5624	0	-1.7582	0	-1.4003	0	-1.7270	0
Switzerland	384	-2.5479	0	-2.5372	0	-0.5586	0	-1.8066	0
United Kingdom	384	-2.2026	1	-2.5383	1	-0.9372	0	-2.5291	1

Note: The ADF test is performed including a constant (ADF_C) and a constant and a time trend in the regression (ADF_C+T). Asymptotic critical values for cointegration are taken from MacKinnon (1996). With constant: -2.87 (5 %), -2.57 (10 %). Constant and time trend: -3.42 (5 %), -3.13 (10 %). Eliott *et al.* (1996) propose a simple modification of the ADF_C (GLS_C) and the ADF_C+T (GLS_C+T) tests in which the data are first detrended, so that explanatory variables are "taken out" of the data prior to running the test regression. Asymptotic critical values for GLS_C are those of ADF t-statistic when there is no constant: -1.94 (5 %), -1.62 (10 %). Asymptotic critical values for GLS_C+T are taken from Eliott *et al.* (1996), Table 1: -2.89 (5 %), -2.57 (10 %). Statistical significance is indicated by a single asterisk (*) for the 10% level, and a double asterisk (**) for the 5% level.

Figure 2. Standardized deviations from long run PPP (relative PPP)

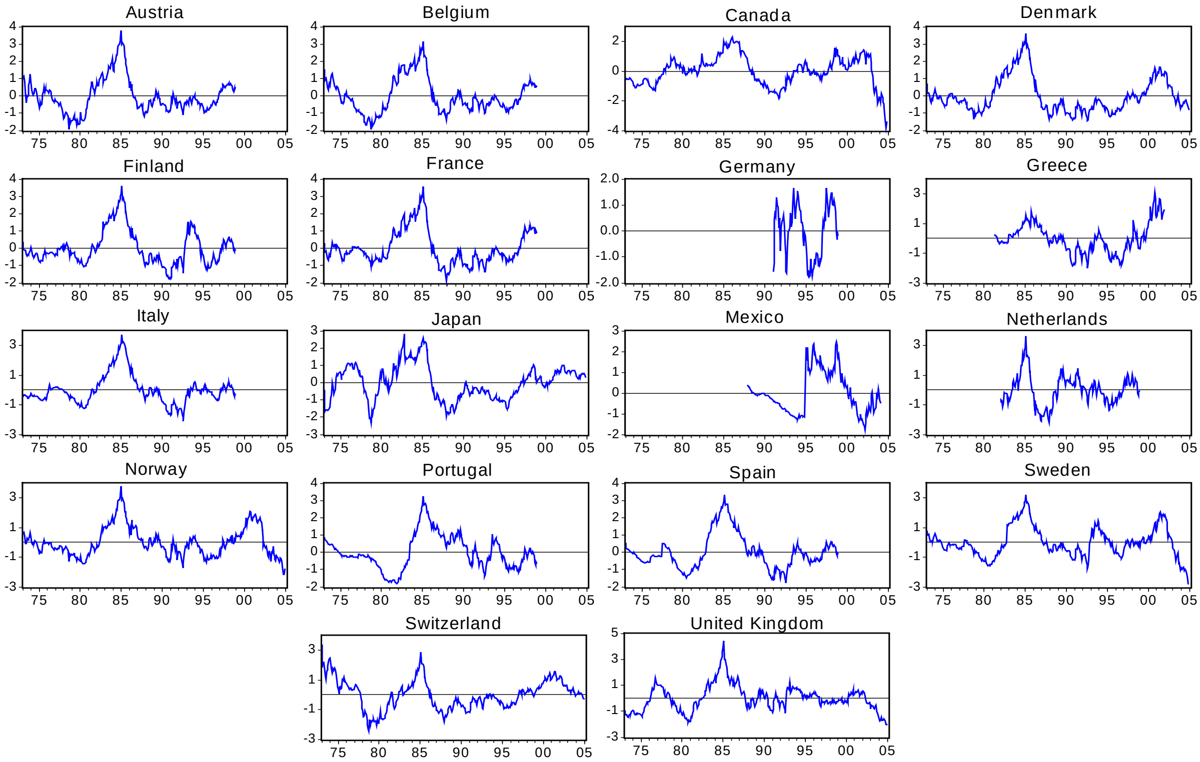

Table 2. Linear cointegration test

	Obs	ADF_C	Lags	ADF_C+T	Lags	GLS_C	lags	GLS_C+T	Lags
Australia	312	-2.3351	2	-2.3085	2	-2.1094	2	-2.1157	2
Belgium	312	-3.0277	13	-3.0021	13	-2.4371	13	-1.7380	2
Canada	383	-1.9073	11	-0.7306	0	-1.7837	11	-2.3538	11
Denmark	384	-2.7199	14	-2.7174	14	-2.6259*	14	-1.9644	2
Finland	312	-1.9053	0	-2.2985	6	-2.3267	6	-2.7619	5
France	312	-2.0000	3	-1.9835	3	-2.0062	3	-1.9093	3
Germany	96	-2.0486	2	-2.0372	2	-2.1487	9	-1.5341	0
Greece	249	-1.9087	0	-1.4121	4	-2.3417	14	-2.7196	13
Italy	312	-2.7336	13	-2.7480	13	-2.7853	13	-2.8156	13
Japan	383	-2.8197	3	-2.8334	3	-2.6489	3	-2.3712	1
Mexico	195	-2.0324	0	-2.0268	0	-1.9114	0	-1.9737	0
The Netherlands	204	-4.5463**	13	-4.6165**	13	-4.5299**	13	-3.6768**	13
Norway	384	-2.1903	2	-2.0094	0	-2.0116	0	-2.1572	0
Portugal	312	-2.3370	7	-2.3308	7	-2.1293	14	-2.1196	14
Spain	312	-1.5480	1	-1.5448	1	-1.4040	0	-1.4146	0
Sweden	383	-1.4150	1	-1.4104	1	-1.5588	1	-1.8023	1
Switzerland	384	-2.8868	2	-2.9082	2	-1.5682	2	-1.7173	7
United Kingdom	384	-1.9742	0	-1.9939	0	-1.5144	0	-2.3174	1

Note: The ADF test is performed including a constant (ADF_C) and a constant and a time trend in the regression (ADF_C+T). Asymptotic critical values for cointegration are taken from Davidson and MacKinnon (1993). With constant: -3.34 (5 %), -3.04 (10 %). Constant and time trend: -3.78 (5 %), -3.50 (10 %). Asymptotic critical values for GLS_C and GLS_C+T are taken from Perron and Rodríguez (2001). Asymptotic critical values for GLS_C are: -2.76 (5 %), -2.47 (10 %). For GLS_C+T are: -3.19 (5 %), -3.07 (10 %). Statistical significance is indicated by a single asterisk (*) for the 10% level, and a double asterisk (**) for the 5% level.

Table 3 Enders-Ludlow test for nonlinear unit root in real exchange rates

	Obs	Lags	k^*	c^*	a^*_1	b^*_1	a_0	$1+r^2/4$	*F_all*	*F_trig*
Austria	312	11	118	-0.0402	-0.0159	0.0725	0.9598	1.0014	7.7429*	7.1254
				(-2.8502)	(-0.8101)	(3.6988)				
Belgium	312	1	5	-0.0287	0.0580	0.0426	0.9713	1.0013	8.1342**	10.9000**
				(-2.2127)	(3.3670)	(2.9276)				
Canada	383	0	3	-0.0302	0.0164	0.0649	0.9698	1.0011	5.8710	8.7123**
				(-2.3104)	(1.0245)	(4.1017)				
Denmark	384	1	6	-0.0131	0.0062	0.0499	0.9869	1.0006	6.5681	8.9923**
				(-1.4337)	(0.5024)	(4.2397)				
Finland	312	1	145	-0.0182	-0.0528	0.0072	0.9818	1.0007	5.6052	6.7200
				(-1.7669)	(-3.6092)	(0.4907)				
France	312	5	140	-0.0120	0.0355	-0.0236	0.9880	1.0005	6.2338	7.8181*
				(-1.6397)	(3.1656)	(-2.2582)				
Germany	96	12	23	-0.1434	-0.0949	-0.1147	0.8566	1.0055	5.1568	3.3257
				(-2.7184)	(-1.6688)	(-1.9836)				
Greece	249	21	8	-0.0235	-0.0299	0.0620	0.9765	1.0012	4.2143	5.6940
				(-1.6701)	(-1.5634)	(3.0221)				
Italy	312	2	122	-0.0082	0.0165	0.0236	0.9918	1.0002	4.2671	5.4260
				(-1.3348)	(1.8679)	(2.6847)				
Japan	383	12	123	-0.0295	-0.0500	0.0278	0.9705	1.0008	7.3892	7.1158
				(-2.7037)	(-3.2782)	(1.8453)				
Mexico	195	1	51	-0.0074	-0.0458	-0.0152	0.9926	1.0006	3.8810	5.5410
				(-0.7216)	(-3.1597)	(-1.0448)				
The Netherlands	204	0	77	-0.0148	0.0440	0.0513	0.9852	1.0011	4.2024	5.5614
				(-1.0358)	(2.1879)	(2.5327)				
Norway	384	0	172	-0.0129	-0.0501	0.0397	0.9871	1.0010	7.1233	9.6609**
				(-1.2705)	(-3.4805)	(2.7614)				
Portugal	312	1	2	-0.0043	-0.0266	0.0071	0.9957	1.0002	5.7056	8.5550**
				(-0.5657)	(-3.7885)	(0.7014)				
Spain	312	1	3	-0.0075	0.0186	0.0225	0.9925	1.0002	3.8852	5.2050
				(-0.9859)	(1.9460)	(2.2911)				
Sweden	383	1	173	-0.0081	-0.0414	0.0025	0.9919	1.0004	3.8831	5.3827
				(-0.9215)	(-3.2794)	(0.2026)				
Switzerland	384	2	101	-0.0224	0.0377	0.0381	0.9776	1.0007	8.1406*	8.9067**
				(-2.5115)	(2.9488)	(2.9989)				
United Kingdom	384	1	173	-0.0160	-0.0290	-0.0283	0.9840	1.0004	5.7240	6.4703
				(-1.9867)	(-2.5938)	(-2.4250)				

Note: The estimated model is:

$$\Delta\hat{e}_t = \left[c^* + a_1^* \ \mathrm{sen}\frac{2\ \pi\ k^*}{T} t + b_1^* \ \cos\frac{2\ \pi\ k^*}{T} t \right] \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_{t-i} \Delta\hat{e}_{t-i} + \varepsilon_t$$

where $\hat{e}_t$ are the residual of the regression of the real exchange rate on $\delta_1+\delta_2 t$. τ-statistics in brackets. Critical values at 90% and 95% are respectively 7.46 and 8.25 for *F_all* statistic (H_0: $c^*=a^*_1=b^*_1=0$), 7.27 and 8.07 for *F_trig* statistic (H_0: $a^*_1=b^*_1=0$) and –3.48 and –3.15 for the τ-statistic (H_0: $c^*=0$). Statistical significance is indicated by a single asterisk (*) for the 10% level and double asterisk (**) for the 5% level.

Table 4. Enders-Ludlow test for nonlinear cointegration

	Obs	Lags	k^*	c^*	a^*_1	b^*_1	a_0	$1+r^2/4$	F_all	F_trig
Austria	312	13	118	-0.0417	-0.0156	0.0671	0.9583	1.0012	6.4428	5.6371
				(-2.7086)	(-0.7602)	(3.2922)				
Belgium	312	13	118	-0.0383	0.0514	-0.0345	0.9617	1.0010	6.9179	5.7027
				(-2.9129)	(2.7617)	(-1.9046)				
Canada	383	0	85	-0.0154	-0.0586	-0.0107	0.9846	1.0009	4.9201	6.4014
				(-1.3032)	(-3.5271)	(-0.6425)				
Denmark	384	13	6	-0.0364	0.0254	0.0531	0.9636	1.0009	7.0267	6.0945
				(-2.8075)	(1.6123)	(3.1890)				
Finland	312	1	118	-0.0272	0.0638	0.0154	0.9728	1.0011	5.9587	6.5060
				(-2.1135)	(3.5208)	(0.8434)				
France	312	7	28	-0.0325	0.0364	0.0586	0.9675	1.0012	5.3428	5.5265
				(-2.1605)	(1.7147)	(2.7853)				
Germany	96	12	28	-0.2235*	-0.0783	0.1465	0.7765	1.0069	6.4982	3.5066
				(-3.5026)	(-1.2381)	(2.2685)				
Greece	249	13	115	-0.0780*	0.1471	0.0133	0.9220	1.0055	12.4052**	10.1366**
				(-3.5730)	(4.5017)	(0.3976)				
Italy	312	0	118	-0.0220	0.0680	0.0235	0.9780	1.0013	6.7518	8.2287**
				(-1.7586)	(3.8238)	(1.3380)				
Japan	383	1	173	-0.0237	-0.0078	-0.0476	0.9763	1.0006	5.3982	5.4185
				(-2.2583)	(-0.5219)	(-3.2298)				
Mexico	195	11	94	-0.0682	-0.2218	0.0027	0.9318	1.0123	19.0026**	24.0649**
				(-2.5592)	(-6.8971)	(0.0752)				
The Netherlands	204	13	47	-0.1517**	0.1285	-0.0455	0.8483	1.0046	11.4602**	6.0393
				(-4.4343)	(3.2701)	(-1.1034)				
Norway	384	4	6	-0.0413	0.0481	0.0720	0.9587	1.0019	7.6701	8.9860**
				(-2.4398)	(2.3048)	(3.4938)				
Portugal	312	22	15	-0.0353	0.0159	0.0627	0.9647	1.0010	5.1755	5.1539
				(-2.1444)	(0.7919)	(3.1647)				
Spain	312	1	4	-0.0325	-0.0518	-0.0029	0.9675	1.0007	4.3450	5.0212
				(-2.1151)	(-2.9797)	(-0.1496)				
Sweden	383	1	6	-0.0307	0.0486	0.0289	0.9693	1.0008	5.1204	5.5765
				(-2.5273)	(2.8835)	(1.8597)				
Switzerland	384	7	6	-0.0508	0.0452	0.0789	0.9492	1.0021	8.3724**	9.4044**
				(-3.0233)	(2.1957)	(3.9527)				
United Kingdom	384	1	22	-0.0305	0.0597	0.0415	0.9695	1.0013	7.3793	7.7591*
				(-2.2661)	(3.1565)	(2.2583)				

Note: The estimated model is:

$$\Delta\hat{e}_t = \left[c^* + a_1^* \text{ sen} \frac{2\,\pi\,k^*}{T} t + b_1^* \cos\frac{2\,\pi\,k^*}{T} t \right] \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_{t-i} \Delta\hat{e}_{t-i} + \varepsilon_t$$

where $\hat{e}_t$ are the long-run regression detrended residuals [model (6) in text]. τ-statistics in brackets. Critical values at 90% and 95% are, respectively 8.20 and 9.06 for *F_all* statistic (H_0: $c^*=a^*_1=b^*_1=0$), 7.23 and 8.02 for *F_trig* statistic (H_0: $a^*_1=b^*_1=0$), and –3.84 and –3.5 for the τ-statistic (H_0: $c^*=0$). Statistical significance is indicated by a single asterisk (*) for the 10% level, and a double asterisk (**) for the 5% level.

Table 5. Vector error correction model with Fourier Decay

	β_0	β_1	β_2	Lags	α_{11}	α_{12}	α_{13}	α_{21}	α_{22}	α_{23}	α_{31}	α_{32}	α_{33}
Austria	-0.0615	1.0505	-0.2719	12	-0.0556***	-0.0122	0.0683***	0.0002***	0.0046**	0.0029	-0.0005	-0.0004	-0.0017
					(-3.0163)	(-0.5787)	(3.2162)	(2.6056)	(2.2781)	(1.4484)	(-0.3973)	(-0.2828)	(-1.2583)
Belgium	0.0447	1.4930	-0.5167	7	-0.0398***	0.0592***	-0.0302*	0.0032	0.0014	0.0002	-0.0027***	-0.0003	-0.0014
					(-2.4473)	(3.3576)	(-1.7090)	(0.9149)	(0.8449)	(0.1418)	(-2.4090)	(-0.2867)	(-1.1598)
Canada	-1.6283	-0.8508	1.3347	1	-0.0207*	-0.0558***	-0.0142	-0.0007	-0.0006	-0.0028	-0.0010	0.0020	-0.0033
					(-1.7449)	(-3.4183)	(-0.8577)	(-0.2048)	(-0.1484)	(-0.6588)	(-0.4359)	(0.6699)	(-1.0704)
Denmark	0.8261	0.8966	-0.5840	7	-0.0241*	0.0241	0.0554***	0.0046	-0.0029	-0.0011	-0.0028***	0.0006	-0.0013
					(-1.7434)	(1.4846)	(3.2910)	(-1.3376)	(-1.1417)	(-0.4191)	(-2.6723)	(0.4888)	(-1.0005)
Finland	-0.8691	-0.2118	0.8589	13	-0.0512***	0.0642***	0.0086	0.0007	-0.0024	-0.0029	0.0002	0.0000	-0.0018
					(-3.0230)	(3.3686)	(0.4468)	(-1.1071)	(-0.9837)	(-1.1626)	(0.1858)	(-0.0098)	(-1.3614)
France	1.6086	2.0800	-2.0346	7	-0.0566***	0.0278	0.0667***	0.0006**	0.0027*	0.0007	-0.0042***	0.0044***	0.0001
					(-3.0368)	(1.2580)	(3.0075)	(2.0944)	(1.7659)	(0.4326)	(-3.2682)	(2.9320)	(0.0797)
Germany	30.7185	-0.2687	-7.5301	2	-0.0483	-0.0914	0.0833	0.0031	-0.0030	0.0113	-0.0067***	-0.0004	0.0033
					(-0.9523)	(-1.3893)	(1.2751)	(-0.5125)	(-0.3948)	(1.5175)	(-2.9681)	(-0.1387)	(1.1470)
Greece	8.3907	0.4543	-1.4297	12	-0.0544***	0.1410***	0.0126	0.0087*	0.0091	-0.0042	-0.0018	0.0008	-0.0023
					(-2.3551)	(4.3441)	(0.3748)	(1.9061)	(1.3577)	(-0.6068)	(-1.2016)	(0.3578)	(-1.0599)
Italy	6.8652	1.1196	-0.9052	8	-0.0321**	0.0704***	0.0218	0.0028	-0.0007	0.0004	-0.0019	-0.0007	-0.0014
					(-2.0213)	(3.7165)	(1.1568)	(-0.3695)	(-0.3100)	(0.1762)	(-1.6338)	(-0.4958)	(-1.0170)
Japan	6.8199	0.0769	-0.4192	13	-0.0315***	-0.0057	-0.0431***	0.0001	-0.0010	-0.0016	-0.0006	0.0012	-0.0016
					(-2.6533)	(-0.3628)	(-2.7527)	(-0.6422)	(-0.4840)	(-0.7883)	(-0.7650)	(1.1173)	(-1.5424)
Mexico	12.1982	1.2590	-3.4785	3	0.0053	-0.1994***	0.0386	0.0120	-0.0095***	0.0047	-0.0049***	-0.0010	0.0016
					(0.1501)	(-6.1066)	(1.0914)	(-2.2447)	(-2.4420)	(1.1126)	(-2.3860)	(-0.5093)	(0.7758)
The Netherlands	-9.1930	4.3860	-1.9342	7	-0.0634**	0.1036***	-0.0064	0.0019***	-0.0063**	-0.0007	0.0009	0.0022	-0.0050**
					(-2.1847)	(2.6418)	(-0.1522)	(-2.9344)	(-2.1712)	(-0.2273)	(0.5709)	(1.0614)	(-2.2755)
Norway	0.7693	0.2148	0.0631	12	-0.0565***	0.0515***	0.0701***	0.0054	-0.0013	0.0066**	0.0005	-0.0012	-0.0007
					(-2.8807)	(2.3850)	(3.1788)	(-0.4907)	(-0.4460)	(2.2172)	(0.3096)	(-0.6943)	(-0.4126)
Portugal	1.7915	1.1128	0.0006	7	-0.0367***	0.0215	0.0400**	0.0117	-0.0025	0.0001	-0.0013	0.0017	-0.0004
					(-2.3825)	(1.1151)	(2.1226)	(-0.4544)	(-0.3630)	(0.0213)	(-1.2472)	(1.2601)	(-0.3001)
Spain	2.8025	0.7702	-0.1584	1	-0.0380***	-0.0586***	-0.0073	0.0062	0.0034	-0.0065	-0.0031***	-0.0021	-0.0003
					(-2.3600)	(-3.2246)	(-0.3732)	(1.0109)	(0.8959)	(-1.6202)	(-2.3907)	(-1.4186)	(-0.1790)
Sweden	-0.9339	-0.3680	1.0352	8	-0.0406***	0.0528***	0.0337**	0.0043	-0.0015	0.0021	-0.0003	-0.0004	0.0003
					(-2.9456)	(3.1065)	(1.9701)	(-0.5699)	(-0.4619)	(0.6660)	(-0.2826)	(-0.3059)	(0.2323)
Switzerland	3.7678	-0.2588	-0.5365	13	-0.0566***	0.0439**	0.0756***	-0.0011	0.0013	-0.0001	-0.0013	0.0002	-0.0005
					(-3.0300)	(2.0826)	(3.6206)	(0.7740)	(0.6855)	(-0.0461)	(-1.1126)	(0.1253)	(-0.3545)
United Kingdom	-1.7666	0.3859	-0.0363	13	-0.0589***	0.0722***	0.0414**	0.0041	0.0032	-0.0016	-0.0001	0.0011	-0.0014
					(-3.3625)	(3.6429)	(2.1405)	(1.1507)	(1.0170)	(-0.5092)	(-0.0599)	(0.7585)	(-1.0124)

Note: t-statistics in brackets. Statistical significance is indicated by a single asterisk (*) for the 10% level, a double asterisk (**) for the 5% level and a triple asterisk (***) for the 1% (H_0: $\alpha_{ij}=0$). The long-run estimated model is $s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + e_t$ and the non-linear vector error correction model is:

$$\begin{pmatrix} \Delta s_t \\ \Delta p_t \\ \Delta p_t^* \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} 1 \\ \text{sen}\,\frac{2\pi k^* t}{T} \\ \cos\frac{2\pi k^* t}{T} \end{pmatrix} \hat{e}_{t-1} + \sum_{i=1}^{p-1} \Phi_i \begin{pmatrix} \Delta s_{t-i} \\ \Delta p_{t-i} \\ \Delta p_{t-i}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}$$

Table 6. Short term linear Granger causality test

		Δs_t *equation*		Δp_t *equation*		Δp_t^* *equation*	
	DF	Δp_t ***lags***	Δp_t^* ***lags***	Δs_t ***lags***	Δp_t^* ***lags***	Δs_t ***lags***	Δp_t ***lags***
Austria	12	10.4364	12.4726	12.0429	22.7883**	12.5714	25.5890**
		(0.5777)	(0.4085)	(0.4422)	(0.0296)	(0.4009)	(0.0123)
Belgium	7	10.2437	9.1171	7.0603	15.0159**	13.5425*	14.1529**
		(0.1752)	(0.2444)	(0.4226)	(0.0358)	(0.0599)	(0.0485)
Canada	1	0.0009	0.5280	0.6653	48.1249***	0.7361	6.7112***
		(0.9758)	(0.4675)	(0.4147)	(0.0000)	(0.3909)	(0.0096)
Denmark	7	3.3708	8.1882	10.1763	27.0576***	14.6626**	25.3063***
		(0.8487)	(0.3163)	(0.1788)	(0.0003)	(0.0406)	(0.0007)
Finland	13	4.1943	12.0235	13.5582	39.8467***	8.0693	24.2404**
		(0.9889)	(0.5257)	(0.4057)	(0.0001)	(0.8391)	(0.0290)
France	7	14.4761**	14.5848**	4.4082	13.3896*	3.2860	28.1031***
		(0.0433)	(0.0417)	(0.7317)	(0.0632)	(0.8573)	(0.0002)
Germany	2	8.2764**	0.0767	1.0555	1.1575	3.8728	23.4262***
		(0.0160)	(0.9624)	(0.5899)	(0.5606)	(0.1442)	(0.0000)
Greece	12	5.4709	14.1380	15.8161	13.7377	15.9779	27.6096***
		(0.9404)	(0.2920)	(0.1998)	(0.3178)	(0.1923)	(0.0063)
Italy	8	12.0587	12.0773	17.8346**	34.9693***	7.3171	18.3472**
		(0.1486)	(0.1478)	(0.0225)	(0.0000)	(0.5028)	(0.0188)
Japan	13	14.3315	20.2634*	13.6693	35.9916***	17.7650	24.5333**
		(0.3509)	(0.0888)	(0.3975)	(0.0006)	(0.1666)	(0.0266)
Mexico	3	1.2316	2.0795	66.6033***	20.6716***	2.8330	9.7949**
		(0.7454)	(0.5561)	(0.0000)	(0.0001)	(0.4181)	(0.0204)
The Netherlands	7	0.9073	2.4245	15.6467**	22.1023***	2.8707	32.0213***
		(0.9962)	(0.9327)	(0.0285)	(0.0024)	(0.8967)	(0.0000)
Norway	12	10.1046	19.0815*	19.1724*	29.0365***	12.6080	27.4672***
		(0.6068)	(0.0866)	(0.0845)	(0.0039)	(0.3982)	(0.0066)
Portugal	7	24.2257***	9.9693	3.7814	26.5614***	16.7535**	17.2182**
		(0.0010)	(0.1903)	(0.8046)	(0.0004)	(0.0191)	(0.0160)
Spain	1	1.5717	2.5580	2.3585	14.0030***	1.1018	11.2725***
		(0.2100)	(0.1097)	(0.1246)	(0.0002)	(0.2939)	(0.0008)
Sweden	8	16.8748**	10.9512	12.5702	33.1962***	8.7641	20.0828***
		(0.0314)	(0.2045)	(0.1275)	(0.0001)	(0.3626)	(0.0100)
Switzerland	13	4.3132	25.3705**	31.2903***	21.0874*	12.4937	15.5409
		(0.9874)	(0.0206)	(0.0031)	(0.0712)	(0.4876)	(0.2748)
United Kingdom	13	8.3402	19.5870	6.6199	25.9363**	14.0200	40.4531***
		(0.8208)	(0.1060)	(0.9207)	(0.0173)	(0.3724)	(0.0001)

Note: estimated equation is:

$$\begin{pmatrix}\Delta s_t\\ \Delta p_t\\ \Delta p_t^*\end{pmatrix}=\begin{pmatrix}m_1\\ m_2\\ m_3\end{pmatrix}+\begin{pmatrix}\alpha_{11}&\alpha_{12}&\alpha_{13}\\ \alpha_{21}&\alpha_{22}&\alpha_{23}\\ \alpha_{31}&\alpha_{32}&\alpha_{33}\end{pmatrix}\begin{pmatrix}1\\ \text{sen}\,\frac{2\pi k^* t}{T}\\ \cos\frac{2\pi k^* t}{T}\end{pmatrix}\hat{e}_{t-1}+\sum_{i=1}^{p-1}\Phi_i\begin{pmatrix}\Delta s_{t-i}\\ \Delta p_{t-i}\\ \Delta p_{t-i}^*\end{pmatrix}+\begin{pmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\\ \varepsilon_{3t}\end{pmatrix}$$

p-values in brackets. Statistical significance is indicated by a single asterisk (*) for the 10% level, double asterisk (**) for the 5% level and a triple asterisk (***) for the 1%.