Transverse angular shift in the reflection of light beams

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Abstract

The reflection of a light beam onto a planar interface can be out of the incidence plane due to the mismatching between the symmetries of the beam and the reflection geometry. The state of polarization of the beam is also involved in the description of the transverse angular shift. The moment characterization of laser beams is used, along with the power expansion of the reflection coefficients and the geometric factors, to find this transverse angular shift.

1. Introduction

The reflection of laser beams at dielectric and metallic interfaces is a very common phenomenon in practical systems handling and transforming laser beams. The analysis of a dielectric/dielectric interface has been deeply treated by Porras [1,2] restricted to the plane of incidence. He uses the characteristic parameters of a light beam defined in terms of the moments of the beam. In this paper we apply a similar procedure to the calculation of the transverse angular shift. This effect means a reflection out of the plane of incidence. The transverse angular shift was outlined and demonstrated in the 70’s for total internal reflection [3–5]. Hugonin and Petit also applied a moment calculation to obtain the displacement of the center of gravity of the beam. A few years ago, Nasalski presented a characterization of the non-specular effects in the reflection of light beams [6]. His treatment is based on a detailed analysis of the changes produced in the reflection. Some of the results of our contribution will be compared with Nasalski’s calculation.

To characterize the beam we calculate the moments of the intensity distribution of its plane wave decomposition. The transformation of the moments by the reflection is calculated by using a Taylor expansion of the reflection coefficients and by taking into account the geometry of the incidence and the state of polarization of the incoming beam. The reflected amplitude for a dielectric/metallic plane interface is obtained using the complex nature of the reflection coefficients. Once the moments of the reflected beam are obtained, the angular shifts along the plane of incidence and along a perpendicular direction are calculated. The results show that an out-of-plane reflection is obtained when the symmetry of the beam does not match with the symmetry of the reflection geometry. It also depends on the state of polarization of the beam. Along the paper we will assume that the beam is paraxial. This means that it extends within a small region around the principal direction of propagation.
In Section 2 we first describe how the three-dimensional beam is transformed by a plane interface. We calculate the transformation by decomposing the beam into plane waves and by using the reflection coefficients. A detailed analysis of the geometry of the incidence will be necessary to correctly apply the reflection coefficients. Also the state of polarization will play an important role in the calculation. Section 3 is devoted to the change of the moments characterizing the beam. A Taylor expansion until second order is applied around the axis defined by the center of the incoming beam. In Section 4 we calculate the transverse angular shift with the general equation obtained to transform the moments. Some cases are treated numerically in this section to show up the influence of the symmetry of the beam, its orientation, its modal character, and the state of polarization. Finally, Section 5 summarizes the main conclusions of the paper.

2. Transformation of the amplitude distribution

A given laser beam can be described by its electric field. The vectorial character is relevant to find its components along the incidence and perpendicular planes. If we assume that the laser propagates along the Z-axis, the electric field at a given z plane is given by its amplitude distribution in that plane, \( \Psi(x, y) \), and by its polarization state. In this paper we have dropped the time dependence because we are primarily interested in the spatial and angular characterization. The amplitude distribution is expanded as a sum of plane waves whose amplitudes are given by the Fourier transform of the amplitude field distribution, \( \Phi(\xi, \eta) \). This angular spectral distribution, the angular spectrum, also carries information of the polarization state of the beam.

If the beam propagates around its main direction inside a paraxial angular region, the magnitude of the plane wave decomposition within a homogeneous dielectric is expressed as follows,

\[
\Phi(\xi, \eta, z) = \Phi(\xi, \eta) \exp \left( \frac{2\pi}{\lambda} z \left[ 1 - \frac{1}{2} \lambda^2 (\xi^2 + \eta^2) \right] \right),
\]

where \( \lambda \) is the wavelength within the homogeneous dielectric media, and \( \xi \) and \( \eta \) are the spatial frequencies along the X and Y directions respectively. The direction of propagation for each plane wave is given by the unitary vector

\[
\hat{r} = \left( \lambda \xi, \lambda \eta, \sqrt{1 - (\lambda^2 \xi^2 + \lambda^2 \eta^2)} \right).
\]

The interface is characterized by the index of refraction in both sides of the interface and by its unitary normal vector \( \hat{N} \),

\[
\hat{N} = (- \sin \varepsilon_0, 0, - \cos \varepsilon_0).
\]

where \( \varepsilon_0 \) is the angle of incidence of the center of the incoming beam. In the reflection, the amplitude of each plane wave changes according with the value of the reflection coefficient, \( r \). Depending on the polarization state of the incoming wavefront, it will be necessary to use both the parallel and the perpendicular reflection coefficients, or either one. These coefficients are given by the Fresnel formulae in terms of the indexes of refraction and the angle of incidence of the incoming plane wave [7]. The incidence angle for each plane wave is given as: \( \varepsilon(\xi, \eta) = \cos^{-1}(\hat{N} \cdot \hat{r}) \). To obtain the reflected plane wave for each incoming component it will necessary: (i) to apply the vectorial reflection law to know the direction of the reflected plane wave, and (ii) to multiply the incident amplitude distribution by the reflection coefficient to obtain the amplitude of the reflected plane wave. The vectorial form of the reflection law [8] provides the following expression for the reflected vector:

\[
\hat{R} = \left( \lambda \xi \cos 2\varepsilon - \sin 2\varepsilon \varepsilon_0 \sqrt{1 - \lambda^2 \xi^2 - \lambda^2 \eta^2} \cdot \varepsilon, \right.


\[
- \lambda \xi \sin 2\varepsilon_0 - \cos 2\varepsilon_0 \varepsilon_0 \sqrt{1 - \lambda^2 \xi^2 - \lambda^2 \eta^2} \right).
\]

Once the direction of the reflection is known by using the previous equation, it is possible to transform the incoming plane wave into the reflected one by changing the coordinate system and modifying the amplitude by means of the reflection coefficients. The reference system for the incident beam was chosen in such a way that Eq. (1) describes the beam (see Fig. 1). This means that the Z axis represents the straight line where the center of gravity of the
beam lies. Then, the first order moments of the intensity distribution of the angular spectrum are zero for the incident beam. Besides, the reference system is aligned with the plane of incidence. The XZ plane defines the main plane of incidence, i.e., the plane of incidence for the center of the beam. This choice means that the projection on the XZ plane of the electric field of the plane wave that propagates within this plane is the parallel component for the center of the beam.

The reflected beam is described with a reference system that is the mirror reflection of the one used for the incident beam. Therefore, the \( Z \) axis remains close to the main direction of propagation of the beam, and \( X' \) and \( Y' \) represent the coordinates on the transversal plane. The actual direction of the propagation of the reflected beam is given by the first order moments of its plane-wave decomposition. The departure of this actual direction from the \( Z \) axis defines the angular shifts. The incident and the reflected reference systems have opposite characters: one of them is dextro and the other is levo. However, this can be taken into account if necessary. The relation between the incident and reflected reference systems makes the coordinate transformation very simple: \( (x', y') = (x, y) \), and in the spatial-frequency domain \( (\xi', \eta') = (\xi, \eta) \).

Fig. 1. Geometry of the coordinate systems for the description of the incident and reflected beams. The coordinate system for the reflection is the mirror image of the coordinate system for the incident beam.

The electric field vector for any incoming component of the plane wave decomposition can be written as,

\[
\Phi'((\xi, \eta)) = \Phi'_\parallel((\xi, \eta))\hat{u}_\parallel + \Phi'_{\perp}((\xi, \eta))\hat{u}_{\perp}
\]

In this equation \( \hat{u}_\parallel \) and \( \hat{u}_{\perp} \) are two unitary vectors along the parallel and perpendicular directions respectively. On the other hand, \( \hat{u}_x \) and \( \hat{u}_y \) are the unitary vectors along the axes of reference of the transversal plane \( XY \). In general \( \hat{u}_x \) and \( \hat{u}_y \) do not coincide. Therefore, the decomposition in parallel and perpendicular direction becomes a key issue that is analyzed as follow. For every plane wave of the angular spectrum it is possible to define a local incidence plane. This plane is defined by the incident and normal vectors (Eqs. (2) and (3)). The cross product of these two vectors defines a vector perpendicular to the local incidence plane. This vector is given as \( \mathbf{v} = \mathbf{I} \times \mathbf{N} \). The projection of this vector on the transversal plane, \( XY \), defines the direction of the perpendicular component of the field. This direction is given by the \( v_x \) and \( v_y \) components of the vector \( \mathbf{v} \) that can be written as,

\[
v_x = -\lambda \eta \cos \varepsilon_0 \, , \quad v_y = \lambda \xi \cos \varepsilon_0 - \sqrt{1 - (\lambda \xi)^2 - (\lambda \eta)^2} \sin \varepsilon_0 \, .
\]

By using these two components we define an angle, \( \beta \), that represents the local rotation of the parallel and perpendicular directions with respect to the main parallel and perpendicular directions. These main directions coincide with the \( X \) and \( Y \) directions respectively. The angle will be given as

\[
\beta = -\tan^{-1}(v_x/v_y) \, .
\]

The reflected angular spectrum can be calculated by locally rotating the incident angular spectrum to meet the actual parallel and perpendicular directions. After this rotation we apply the reflection coefficients, and finally the rotation is undone to retrieve
the correct orientation. This transformation can be written in matricial form as follows,

\[
\begin{pmatrix}
\Phi_i'
\end{pmatrix}
\begin{pmatrix}
\Phi_i'
\end{pmatrix}
- \begin{pmatrix}
\cos \beta & -\sin \beta \\
\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
0 \\
\rho_s
\end{pmatrix}
\begin{pmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
\Phi_i'
\end{pmatrix}
- \begin{pmatrix}
r_\rho \cos \beta + r_\perp \sin \beta \\
(r_\rho - r_\perp) \sin \beta \cos \beta
\end{pmatrix}
\begin{pmatrix}
r_\rho \sin \beta + r_\perp \cos \beta \\
(r_\rho - r_\perp) \sin \beta \cos \beta
\end{pmatrix}
\begin{pmatrix}
\Phi_i'
\end{pmatrix}.
\]

(9)

The last 2 × 2 matrix of the previous equation can be denoted as the reflection matrix. This result coincides with Eq. (61) of Ref. [5].

As we stated at the beginning of this paper, this analysis is valid for light beams whose angular extension is within the paraxial range. This condition is also related with the transversality condition for light beams. This means that once the paraxial condition can be applied then the component of the electric field along the propagation direction at a given point of a curved wavefront can be neglected. Therefore, within the paraxial approach, the electric vector lies on the transversal plane \(XY\).

3. Transformation of the moments

The moments of the intensity distribution of the angular spectrum, \(\mu_{i,j}\), are defined by the following integral,

\[
\mu_{i,j} = \int \int \Phi(\xi, \eta)^{|\xi|^i |\eta|^j} \, d\xi \, d\eta.
\]

(10)

The angular main directions of propagation and the components of the divergence tensor are given by [9,10]

\[
\theta_x = \lambda \mu_{1,0}/\mu_{0,0},
\]

\[
\theta_y = \lambda \mu_{0,1}/\mu_{0,0},
\]

\[
\theta_{x^+} = 4\lambda^2 \left[ \mu_{2,0}/\mu_{0,0} - (\mu_{1,0}/\mu_{0,0})^2 \right],
\]

\[
\theta_{x^-} = 4\lambda^2 \left[ \mu_{1,1}/\mu_{0,0} - (\mu_{1,0}/\mu_{0,0})/\mu_{0,0} \right],
\]

\[
\theta_{y^+} = 4\lambda^2 \left[ \mu_{0,2}/\mu_{0,0} - (\mu_{0,1}/\mu_{0,0})^2 \right],
\]

both for the incident and reflected beam, as a function of the moments of the angular spectrum of the beam. A superscript \(i\) or \(r\) will be added to distinguish between the incident and the reflected beam respectively.

For the incident beam, the electric field is given by Eq. (5) with respect to an orthogonal reference system on the transversal plane. At this point it is necessary to introduce a parameter describing the state of polarization of the incoming wave. This parameter is the following ratio

\[
\chi(\xi, \eta) = \frac{\Phi_i(\xi, \eta)}{\Phi_i(\xi, \eta)}.
\]

(16)

Nasalski defines it in terms of the parallel and perpendicular directions. However, in our case it is more interesting to use the ratio between the \(x\) and \(y\) components. This parameter is complex in nature and will be expressed as \(\chi = |\chi| e^{i\alpha}\). Within the paraxial approach, as Nasalski states, the polarization ratio can be considered constant for the whole beam, \(\chi(\xi, \eta) = \chi_0\). If this condition is not fulfilled then the dependence of \(\chi\) with respect to \((\xi, \eta)\) must be taken into account.

By using the ratio of polarization the squared modulus of the incident plane wave is calculated as

\[
|\Phi_i|^2 = (|\chi|^2 + 1)|\Phi_i|^2,
\]

and therefore there is a relation between the moments of the total intensity and the moments of the intensity of the component along the direction \(Y\). This relation is,

\[
\mu_{j,k} = \frac{\mu_{i,j}}{|\chi_0|^2 + 1},
\]

(18)

where the superindex \(^{i,j}\) means the moment of the component \(y\) of the incident beam. We assume that \(\chi_0\) is constant on the transversal plane.

After some calculation using Eq. (9) the square modulus of the reflected distribution, \(|\Phi_r|^2\), can be written as

\[
|\Phi_r|^2 = \left( |r_\parallel|^2 \cos^2 \beta + |r_\perp|^2 \sin^2 \beta \right)
\]

\[
+ \left( |r_\parallel|^2 + |r_\perp|^2 \sin^2 \beta \cos^2 \beta \right)|\chi_0|^2
\]

\[
+ |r_\parallel|^2 \sin^2 \beta + |r_\perp|^2 \cos^2 \beta
\]

\[
+ \left( |r_\parallel|^2 + |r_\perp|^2 \sin^2 \beta \cos^2 \beta \right)|\chi_0|^2 - |r_\parallel|^2
\]

\[
\times \sin \beta \cos \beta \chi_0 \cos \alpha |\Phi_i|^2,
\]

(19)

where \(\alpha\) is the phase of \(\chi_0\).
As we can see, the result involves not only the reflection coefficients \((r_\perp, r_\parallel)\), but also the geometry of the incidence \((\beta)\), and the polarization state of the beam \((\chi_0)\). These coefficients can be obtained as a function of the spatial frequencies \((\xi, \eta)\). In the paraxial regime the values of the reflection coefficients for the whole beam are around the values of the coefficient for the center of the beam, i.e., \(\xi = \eta = 0\), that incides at an angle \(\varepsilon_0\). Then, after expanding in powers, we write the reflection coefficient as:

\[
r(\xi, \eta) = r^{00} + r^{10}_\xi \xi + r^{01}_\eta \eta + \frac{1}{2} \left[ r^{20}_\xi \xi^2 + 2 r^{11}_\xi \xi \eta + r^{02}_\eta \eta^2 \right] + \cdots ,
\]

where the superscripts indicate the order of the derivation with respect to the variables \((\xi, \eta)\). Besides, the derivatives must be evaluated at \(\xi = \eta = 0\). In the previous equation we dropped the subscripts \(||\) and \(\perp\) because the expansion is done for both coefficients. After some calculus it is found that, within our reference system, the first derivative of \(r\) with respect to \(\eta\) is zero, both for the parallel and the perpendicular directions. This fact is a consequence of the geometry of the problem. Given an incidence angle, \(\varepsilon_0\), if we take another incidence direction \(\xi\), the reflection coefficient will vary in a symmetric way with respect to the value obtained inside the main incidence plane. Therefore, the dependence of \(r\) with respect to \(\eta\) reaches a stationary point at \(\eta = 0\). Then, within the second order approach given in the previous equation, there are only four coefficients different from zero. These coefficients are: \(r^{00}, r^{10}, r^{20},\) and \(r^{02}\). This expansion is also referred in Nasalski’s paper in Eqs. (27.a) and (27.b). However, he uses an exponential form for the reflection coefficients that is more adapted to his formalism.

The situation of the local plane of incidence and the decomposition onto the parallel and perpendicular local directions are characterized by the angle \(\beta\). This angle was defined in the previous section (Eq. (8)). The trigonometric functions that appear in Eq. (19) can be also expanded until second order as follows:

\[
\cos^2 \beta = 1 - 2 \Lambda^2 \eta^2, \tag{21}
\]
\[
\sin^2 \beta = \Lambda^2 \eta^2, \tag{22}
\]
\[
\cos \beta \sin \beta = -\Lambda \eta, \tag{23}
\]
\[
\cos^2 \beta \sin^2 \beta = \Lambda^2 \eta^2, \tag{24}
\]

where \(\Lambda = \lambda / \tan \varepsilon_0\). These equations also provide another restriction to the expansion. The angle of incidence must be large enough to allow the approach. For example, if the incidence is normal, the angle \(\beta\) varies from \(-\pi/2\) to \(\pi/2\) interchanging the role of the parallel and perpendicular components along the way. In this case the approximation clearly fails. Therefore, we must assure that the transverse extension of the beam is small compared with the actual value of the incidence angle. The transverse extension of the incident beam is characterized by the parameter \(\theta_{1r}\). Then, this condition can be written as:

\[
\theta_{1r} \ll \varepsilon_0. \tag{25}
\]

Also the polarization ratio could be expanded in powers of the spatial frequencies if necessary.

At this point, and by using these previous equations it is possible to calculate the moments of the reflected beam in terms of the moments of the incident beam. After applying the previous expansions for \(r_\perp, r_\parallel\), and the trigonometric functions, the moments of the reflected beam are as follows:

\[
\mu_{ij,k} = \frac{1}{\left| \chi_0 \right|^2 + 1} \sum_{l+m=0}^{N} a_{k,l} \mu_{j+l,k+m}, \tag{26}
\]

where the coefficients are given by:

\[
a_{0,0} = \left( r_{\perp}^{00} \right)^2 \left| \chi_0 \right|^2 + \left| r_{\parallel}^{00} \right|^2, \tag{27}
\]
\[
a_{1,0} = \left( r_{\perp}^{00} r_{\parallel}^{10} + r_{\parallel}^{00} r_{\perp}^{01} \right) \left| \chi_0 \right|^2 + \left( r_{\perp}^{00} r_{\perp}^{10} + r_{\parallel}^{00} r_{\parallel}^{01} \right), \tag{28}
\]
\[
a_{0,1} = -\left( \left| r_{\parallel}^{00} \right|^2 - \left| r_{\perp}^{00} \right|^2 \right) \Lambda | \chi_0 | \cos \alpha, \tag{29}
\]
\[
a_{2,0} = \left[ \frac{1}{2} \left( r_{\parallel}^{00} r_{\parallel}^{20} + r_{\parallel}^{02} r_{\parallel}^{00} \right) + \left| r_{\parallel}^{10} \right|^2 \right] | \chi_0 |^2 + \left[ \frac{1}{2} \left( r_{\perp}^{00} r_{\perp}^{20} + r_{\perp}^{02} r_{\perp}^{00} \right) + \left| r_{\perp}^{10} \right|^2 \right], \tag{30}
\]
\[
a_{1,1} = \left( r_{\perp}^{00} r_{\perp}^{10} + r_{\parallel}^{00} r_{\parallel}^{01} \right) - \left( r_{\perp}^{00} r_{\parallel}^{10} + r_{\parallel}^{00} r_{\perp}^{01} \right) \Lambda | \chi_0 | \cos \alpha, \tag{31}
\]
\[
a_{2,0} = \left[ \frac{1}{2} \left( r_{\parallel}^{00} r_{\parallel}^{02} + r_{\parallel}^{02} r_{\parallel}^{00} \right) + \Lambda^2 \left| r_{\parallel}^{00} \right|^2 - \left| r_{\parallel}^{00} \right|^2 \right] \left| \chi_0 \right|^2 + \left[ \frac{1}{2} \left( r_{\perp}^{00} r_{\perp}^{02} + r_{\perp}^{02} r_{\perp}^{00} \right) + \Lambda^2 \left| r_{\perp}^{00} \right|^2 - \left| r_{\perp}^{00} \right|^2 \right]. \tag{32}
\]
The angular characteristics of the reflected beam can be obtained from the results of Eq. (26). These angular parameters were defined in Eqs. (11)-(15).

4. Transverse angular shift

One of the most interesting consequences of the calculation presented previously is that it predicts a reflection outside of the plane of incidence. Transverse angular shifts have been already described [6] and demonstrated [3] by analyzing the three-dimensional propagation of the electromagnetic field along the interface of two dielectric media [11–13]. In the present paper this result is obtained by calculating the characteristic parameters of the reflected beam in three dimensions, extending the Porras’ analysis who calculated inside the plane of incidence [1,2]. Based on the transformation of plane waves by means of the reflection coefficients, our approach allows to describe both the dielectric/dielectric interface partial reflection, and the dielectric/metallic interface reflection.

The transverse angular shift is given by Eq. (12). To better calculate it, we will assume that the incident beam is aligned with the Z axis. This means that $\mu_{1,0} = \mu_{0,1} = 0$, and therefore $\theta_y' = \theta_z' = 0$. After some substitutions using Eqs. (27)–(32) and Eqs. (13)–(15), we find the following results for $\theta_y'$ within the zero, first, and second order of approximation in the calculation of the moments of the reflected beam ($N = 0,1,$ and $2$ respectively).

$$\theta_y'(0) = \theta_y' = 0,$$

$$\theta_y'(1) = \frac{1}{4\lambda} \left( a_{1,0} \theta_{y/2} + a_{0,1} \theta_{y/2} \right),$$

$$\theta_y'(2) = \lambda \left( \frac{a_{1,0} \theta_{y/2}^2 + a_{0,1} \theta_{y/2}^2 + \mu_{1,1} \theta_{y/2} + \mu_{0,2} \theta_{y/2} + a_{0,2} \theta_{y/2} + a_{2,0} \theta_{y/2}} {a_{1,1} \theta_{y/2} + a_{1,2} \theta_{y/2} + \mu_{1,0} \theta_{y/2} + a_{0,0} \theta_{y/2} + a_{0,2} \theta_{y/2}} \right).$$

Fig. 2. Plot of the $a_{i,j}$ coefficients, normalized to the factor $(|\chi_0|^2 + 1)$, as a function of the incidence angle $\varepsilon_0$. The plots are for the case of an Air/Al plane interface. Each plot contains several cases of linear or elliptic polarizations.
The zeroth order approach produces a null transverse shift. For the first order approach we already find a non null transverse shift that depends on the elements of the divergence tensor. The dependence involves higher order moments (until 3rd order) if the approach is extended to the 2nd order. Eq. (34) can be compared with Eq. (40) of the Nasalski’s paper. Although Nasalski splits the calculation into the parallel and perpendicular components, the dependence with the angular divergence is also found when the Rayleigh range distance used in his calculations is written in terms of the divergence. The second order approach (Eq. (35)) involves the same kind of dependence but in a little more complicated fashion. In the present paper it is possible to find the transverse angular shift without the parallel and perpendicular decomposition. Actually, the geometry of the incidence and the state of polarization are included in the $a_{i,j}$ coefficients through the Taylor expansion used to obtain the coefficients in terms of $A$ (see Eqs. (21)–(24)) and the ratio of polarization, $\chi$.

4.1. Numerical examples

In this section we perform some numerical calculation of the transverse angular shift in the second order approximation for some few typical examples. These examples involve circular and elliptic Gaussian beams, Hermite–Gaussian modes, and several states of polarization. The calculation wavelength is $\lambda = 633$ nm, and the interfaces are Air/Al ($n_{Al} = 1.44 + i5.23$), Air/Ag ($n_{Ag} = 0.22 + i3.44$), and external and internal reflection in an Air/Glass ($n = 1.5$) interface.

The transformation of the moments depends on the value of the $a_{i,j}$ coefficients. They are related with the values of the refraction indexes involved in

![Fig. 3. Variation of the longitudinal and transverse angular shift as a function of the angle of incidence and the ellipticity of a Gaussian beam for the parallel and perpendicular linear polarizations. The axes of the elliptical intensity distribution are oriented at 45° with respect to the plane of incidence. The value of the orthogonal Gaussian divergences along the axes of the ellipse are: $\theta_1 = 20$ mrad, and $\theta_2 = 20, 13,$ and 10 mrad as indicated in the figure. The direction of the axis described by $\theta_1$ is along the first-third quadrants of the $XY$ plane. The interface is between two dielectric media of index 1.5 and 1 (internal reflection).]
the interface, \( n_i \), \( n_r \), the angle of incidence, \( \varepsilon \), and the state of polarization of the incoming beam defined by \( \chi_0 \). Fig. 2 represents the values of the \( a_{jk} \) coefficients (Eqs. (27)–(32)) as a function of the angle of incidence, \( \varepsilon_0 \), for an Air/Ag plane interface. Actually, the plots include the normalization factor \( 1/|\langle \chi_0 \rangle|^2 + 1 \) that allows the comparison between different states of polarization. Although the numerical values of these coefficients are small, they are multiplied by the values of the moments of the incident beam to obtain the moments of the reflected beam. The moments are calculated in the spatial-frequency domain and they can reach very large values producing a non negligible angular shift. As we will see in the cases presented below, the numerical value of the transverse angular shift is, in certain cases, comparable to the longitudinal angular shift obtained within the plane of incidence.

The symmetry of the incident beam is a critical characteristic to enhance the transverse angular shift. This fact is shown in Fig. 3 where we plotted the angular shifts for elliptic Gaussian beams with different ratio between axes. The elliptical intensity distributions are oriented having their axis 45° apart from the plane of incidence. The case is an internal reflection for a dielectric/dielectric \((n_i/n_r = 1/1.5)\) interface. The beams are linearly polarized along the parallel and perpendicular main directions. These directions coincide with the \( X \) and \( Y \) axes respectively. The plots on the left correspond with the longitudinal angular shift, \( \theta^l \). The plots on the right are for the transverse angular shift, \( \theta^t \). We can see that \( \theta^t \) is about twice larger than \( \theta^l \). The transverse angular shift increases with the ellipticity of the incoming beam, being negligible with respect to \( \theta^l \) for circularly symmetric beams. Our results for \( \theta^t \)

![Graphs showing angular shifts](image)

**Fig. 4.** Variation of the longitudinal and transverse angular shifts for several Gaussian–Hermite modes. The orientation of the axes of the ellipse is 45° with respect to the plane of incidence (as in Fig. 3). The Gaussian divergences of the mode \((0,0)\) are \( \theta_1 = 20 \text{ mrad} \) and \( \theta_2 = 6.7 \text{ mrad} \). The modes represented are \((1,1), (2,2), \) and \((3,3)\), as shown in the figure. The states of polarization are linear along the parallel and perpendicular main directions as indicated in the figures. The interface is Air/Ag.
coincide very well with those presented in Fig. 2 of Ref. [1] taking into account that in our case the divergence is 20 mrad. The signs also change due to the opposite orientation of the X axis.

Beyond the limit angle and within our formalism the \( a_{i,j} \) coefficients until second order are: \( a_{0,0} = |\chi_0|^2 + 1 \), \( a_{1,0} = a_{0,1} = a_{2,0} = a_{1,1} = 0 \), and
\[
a_{0,2} = (|\chi_0|^2 + 1) A^2. \tag{36}
\]
Therefore, after some substitutions, the transverse angular shift for total internal reflection is given by
\[
\theta_y^2(2) = 4\lambda^2 \frac{\mu_{0,3}/\mu_{0,0}}{4\tan^2 x_0 + \theta_y^2}. \tag{37}
\]
This equation means that if the beam is symmetric with respect to the XZ plane then \( \mu_{0,3} = 0 \), and there is not transverse angular shift. Therefore the contribution of the asymmetry of the beam to the effect would be reported by this equation. The beams used in our examples are all symmetric with respect to the XZ. Therefore, the calculated transverse angular shift is zero after the limit angle (see Fig. 3).

The multimodal character of the beam also affects the value of the transverse angular shift. In Fig. 4 we plotted the value of \( \theta_y^2 \) calculated for several Hermite–Gaussian modes inciding onto an Air/Ag interface. The figure shows up that higher modes implies higher angular shifts. Both \( \theta_x^2 \) and \( \theta_y^2 \) are plotted in this figure for linear polarized Hermite–Gaussian beams with elliptic shape oriented at 45° and orthogonal Gaussian divergences of 20 mrad and 6.7 mrad. The state of polarization is linear pointing along the parallel and the perpendicular directions.

In Fig. 5 we have calculated the longitudinal and transverse angular shift for a Gaussian beam with circular shape and divergence of 20 mrad for an

Fig. 5. Variation of the longitudinal and transverse angular shifts as a function of the angle of incidence for a circularly symmetric Gaussian beam having a divergence of 20 mrad. For the figures on the top the polarization is linear oriented at different angles: 0° (parallel), 45°, and 90° (perpendicular). At the bottom, the figures corresponds with elliptical polarization states having \( |\chi_0| = 1 \). The interface is Air/Glass \((n = 1.5)\).
Air/Glass ($n = 1.5$) incidence. The plots in the top of the figure correspond with linear polarization at $0^\circ$ (parallel main direction along the $X$ axis), $45^\circ$, and $90^\circ$ (perpendicular main direction along the $Y$ axis). In these cases the transverse angular shift is about two orders of magnitude smaller than $\theta_r$. The bottom of the figure represents several cases of elliptic polarized light. We fix $|\chi_0| = 1$. Now the longitudinal angular shift is about ten times smaller than for the linear polarization case, and it does not depend on the phase of $\chi_0$.

5. Conclusions

In this paper we have proposed a method for calculating the angular moments of reflected three-dimensional laser beams within the paraxial approach. This method can be applied to dielectric/dielectric and dielectric/metallic reflections. This study completes previous contributions restricted to the plane of incidence. The method uses a power expansion of the reflection coefficients and applies it to the three-dimensional plane-wave decomposition of the incident beam. The geometry of the incidence and the local change of the plane of incidence is incorporated to the calculation. Its dependence with respect to $\xi$ and $\eta$ is also expanded until second order. The polarization state of the beam is assumed to be constant within the paraxial approach.

As a result of the method, we focus on the transverse angular shift. This effect has been described previously within a more pure electromagnetic framework. The same effect is obtained here within the moment characterization of laser beams. The actual value of the transverse shift depends on the matching of the symmetry of the beam and the geometry of the incidence. It also depends on the multimodal character of the beam and the state of polarization of the incoming beam.

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