Stress sensor based on light scattering by an array of birefringent optical waveguides

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Abstract. In this work we propose a new stress-sensing technique based upon measurement of light scattering produced by an array of birefringent waveguides. When external stress is applied to the array of waveguides, their optical properties are modified via the photo-elastic effect. Specifically, the intensity of light scattered by the optical waveguides is significantly affected by stress. Analysis of this change provides a means to assess the strength and direction of the external force.

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Classification numbers: 07.07 Df (sensors (chemical, optical, electrical, movement, gas, etc.); remote sensing); 42.81 i (fiber optics); 42.81 Pa (sensors, gyros); 42.81 Qb (fiber waveguides, couplers and arrays); 42.81 Dp (propagation, scattering and losses; solitons);

1. Introduction
In the two past decades there has been an increasing number of proposals to perform sensor optical waveguides and to characterize significant parameters as temperature tolerance, elasticity, and optical properties as refractive index distribution, as some examples [1]. Regarding to stress-sensing, different kind of optical fiber sensors have been investigated, such as interferometric and grating-based systems. Stress-sensors based on modal analysis have very promising perspectives [2], but this technique requires far-field radiation pattern characterization for particular modal orders of the optical fiber. As we shall see, our scattering-based proposal provides a less complicated alternative. With respect to previous work on light scattering applied to stress-sensing, it is well known that backward scattering data provides information on the refractive index distribution and, therefore, changes appearing in this function by presence of strains in the structure can be detected by analyzing the scattering diagram at 180º [3]. In this paper we propose a more generalized method based upon the analysis of multiple scattering of light by an array of birefringent optical waveguides.
The basis of our method consists in comparing light scattering before and after an external force is applied. In order to compute multiple scattering by an array of birefringent waveguides, a very general theoretical formulation and the corresponding numerical procedures have been required. These powerful computation tools have been developed by the authors in a previous paper [4] and we will apply them here to simulate the stress sensor.

We will consider an external force acting on the array of optical fibers along a given direction. Then we will calculate the new dielectric permittivity tensor via photo-elastic effect and determine the shape of the intensity profile for light scattering. We have observed a variation in this profile compared to multiple scattering in absence of external force action. Hence, the procedure appears sensible to determine the direction and magnitude of the external force and the proportional changes in the dielectric permittivity tensor.

The plan of the paper is as follows. In section 2 we provide an overview of the necessary theoretical background. As mentioned above, the details of the model and numerical methods have been presented somewhere else [4]. Section 3 presents the new stress sensing technique. First, in subsection 3.1, we introduce the physical principle that allows stress sensing. Then, in subsection 3.2, we report and discuss the obtained results that support our proposal. Finally, section 4 is devoted to summarize and draw the main conclusions of this work.

2. Theoretical background

2.1 Light scattering by a cylindrical birefringent waveguide

Let us consider a cylindrical optical waveguide (optical fiber) of radius R placed within an infinite, isotropic, homogeneous medium. Let the fiber axis be the Z-axis of our coordinate system as shown in Figure 1. The symbol \( \rho \) will denote the position of points belonging to the XY plane. In this work we have considered homogeneous birefringent waveguides with uni-axial symmetry, being their optical axis parallel to the Z-direction. Consequently, the dielectric permittivity tensor \( \varepsilon \) inside the fiber is diagonal and has only two independent components: \( \varepsilon_{33} \), associated to propagation along the Z-axis, and \( \varepsilon_{11} \), corresponding to the XY-plane, since \( \varepsilon_{11} = \varepsilon_{22} \).

We have investigated the scattering of a monochromatic linearly polarized light wave propagating along the Z-direction. Let the incident magnetic field of light, \( h^{(0)} \), lie on the XY-plane. In our calculations we have followed the eikonal formulation of light scattering and we have kept the first-order approximation, acceptable for weak scattering processes [5]. Such approach leads to the following integral equation (See ref.[6] for further details):

\[
    h(\rho) = h^{(0)} + \int_{\Omega} d^2 \rho' \left[ \frac{1}{4i} H_0^{(1)}(\rho, \rho') \hat{P}_1(\rho') + \hat{P}_2(\rho, \rho') \right] h^{(0)},
\]

where \( H_0^{(1)} \): Hankel function of the first kind and zero-order; \( \hat{P}_1 \) and \( \hat{P}_2 \): matrix functions depending on the dielectric permittivity tensor and the second-derivatives of \( H_0^{(1)} \) (see ref. [6]). The first term in Eq. (1) is simply the incident wave amplitude \( h^{(0)} \). The second contribution is the scattered field amplitude, a complex quantity that depends on the light and scatterer’s properties.
2.2 Mechanical-optical effects

When an external force is applied to an optical waveguide, an elastic deformation occurs. Then, its birefringence is modified through the so-called photo-elastic effect [7]. The mathematical formulation of this phenomenon can be accomplished by introducing a modified dielectric permittivity tensor \( \tilde{\varepsilon} \) that takes into account the photo-elastic effect through some additional terms in the following way [8]:

\[
\tilde{\varepsilon}_{ik} = \varepsilon_{ik} \delta_{ik} + a_1 u_{ik} + a_2 u_{ij} \delta_{jk}
\]  

(2)

where \( \delta_{ik} \) is the Delta-Kronecker; \( a_1 \) and \( a_2 \) are the elastic-optical constants of the medium; and \( u_{ij} \) are the components of the stress tensor, which are proportional to the transverse force per unit length applied to each waveguide.

2.3 Generalization to an array of birefringent waveguides

Let us consider now an array of parallel cylindrical waveguides. We will apply the sampling theorem [9] to generalize the above obtained single-fiber results to a bundle of \( N \) fibers. Notice that the scattered field amplitude produced by a single cylindrical waveguide shows cylindrical symmetry. Hence, the total field amplitude, \( h_{TOT} \), scattered by an array of fibers of radius \( R \) writes [10]:

\[
h_{TOT} = \frac{\pi}{4} \sum_m \sum_n h_i(n d / 2, m d / 2) \frac{J_1 \left( \frac{2\pi}{d} \sqrt{(x - n d / 2)^2 + (y - m d / 2)^2} \right)}{2\pi / d \sqrt{(x - n d / 2)^2 + (y - m d / 2)^2}},
\]  

(3)

where \( h_i \) is the scattered field amplitude corresponding to each single fiber, and \( J_1 \) is the first-order Bessel function. The summation indices \( n \) and \( m \) label the sampling spatial points, \( x_n = n d / 2, y_m = m d / 2 \), being \( d \) the distance between centers of two adjacent fibers. Depending on the geometrical arrangement of the array, different sampling lattices may be used.

3. Results and discussion

3.1 Physical basis of the stress sensing technique

In this work we are dealing with a uni-axial medium, meaning that it presents two refractive indexes: extraordinary, for light polarized along the optical axis (the Z-axis), and ordinary, for XY-plane polarization. That is, directions X and Y are optically indistinguishable. Hence, components X and Y of our incident light beam will undergo exactly the same scattering process and no difference between both components will be detectable.

When the array of fibers is subjected to stress under an external force or pressure, its index tensor is modified through the photo-elastic effect, as explained above. Consequently, the symmetry of the XY plane is broken because now \( \varepsilon_{11} \neq \varepsilon_{22} \), i.e. both directions X and Y are no longer optically indistinguishable. With regard to light scattering, this results in a change in the relative behaviour of both components of the scattered light field. Given the relative variation in the scattered field amplitude, the induced birefringence \( \varepsilon_{11} - \varepsilon_{22} \neq 0 \) can be directly evaluated, which allows us to calculate the direction and strength of the applied external force through formula (2). This is the basis of our proposal for a novel stress sensor.
3.2 Results and discussion

In order to assess the effect induced on the waveguide by an external force, we have proceeded as follows. First, we have integrated Equation (1) by gaussian quadratures [11] to obtain the light field scattered by a single birefringent waveguide. Then, we have used Equation (3) to compute the total scattering field amplitude corresponding to an array of cylindrical waveguides in the absence of stress. After the application of an external force, the components of the dielectric permittivity tensor have been re-calculated through Equation (2). Finally, we have used this modified tensor $\varepsilon'$ to obtain the new shape of the light scattering intensity profile.

In our calculations we have considered an array of 7 fibers of radius $R=0.75 \, \mu m$, separated by a distance $d=2 \, \mu m$. The birefringent waveguiding medium exhibits ordinary refractive index $n_o=1.34$ and extraordinary refractive index $n_e=1.35$. For the photo-elastic constants $a_1$ and $a_2$, we have considered typical values of silica fibers $a_1 = 0.38 \times 10^{-12} \, m^2/N$, and $a_2 = 2.68 \times 10^{-12} \, m^2/N$.

Without loss of generality, we have chosen an external force applied along the Y-axis. First, we have compared the results obtained under zero external force ($F_y=0$) with the results after application of a very weak force ($F_y=0.01 \, N$). For zero external force, scattering profiles for X-polarized and Y-polarized light are identical because of the axial symmetry of the physical system. However, this symmetry is broken after application of an external force, as stated in subsection 3.1. After stress is induced, the intensity of X-polarized decreases slightly (see Figure 4, dashed-line curve) while Y-polarized light is enhanced (see Figure 4, solid line). Notice that the latter coincides with the direction of application of the external force. This loss of symmetry could be detected by monitoring the light field amplitude after scattering. Therefore, analysis of the scattering profile will allow us to determine the direction of application of the force (i.e. stress). It should be noted that, even for a very weak force $F=0.01 \, N$ the difference in intensity between both polarizations is ~20%.

From Figure 4 we can also state that the major impact of stress occurs at the central peak of the scattered light profile, which corresponds to forward scattering. This is very convenient for practical purposes since forward scattering observation could be easily performed. We have calculated the dependence of forward scattering on the strength of the applied force. Figure 5 shows the forward-scattering intensity for X- (dashed line) and Y- (solid line) polarized light as a function of the strength of the force, $F_y$, in the range $0<F_y<0.1 \, N$. Notice that the difference between both components is enhanced as $F$ is stronger. For highest values of $F_y$, the intensity of X- polarized light is strongly suppressed. Even for moderate values of the force strength, such as $F_y=0.05 \, N$, a difference around 60% between both polarizations is observed. Since the amount of change for both light polarizations is directly related to the strength of the applied force, Figure 5 provides a calibration curve for our stress-sensing system.

4. Conclusions

In this paper we have evaluated the possibility of implementing a stress sensor based on light scattering by an array of birefringent waveguides. To this end we have calculated the impact of stress on the intensity profile of scattered light.

Our results show a significant change in the profile of the field. The change affects especially the central peak of the intensity profile which corresponds to forward scattering. This is very profitable for applications since forward-scattering measurements are more easily accomplished.
One main consequence of stress is the loss of axial symmetry due to the appearance of two privileged directions: the axis parallel to the force (Y-axis) and the perpendicular axis (X-axis). The asymmetry induced by stress between X- and Y- polarized light provides a method to assess the direction of application of the external force.

Also, we have found that the stress-induced difference between X- and Y- polarized light is strongly dependent on the strength of the applied force. We have plotted this dependence for both polarizations, thus obtaining a “calibration curve” for our stress-sensing system.

Finally, the technique offers good sensitivity, since even for very weak forces (F=0.01 N) a significant difference of ~20% between X- and Y- polarized light can be observed.

We conclude that the effect of stress on scattering by an array of birefringent waveguides could be the basis of an optical stress sensor.

Acknowledgements

The authors gratefully acknowledge financial support from the “New Del Amo foundation” under New Del Amo Joint Academic Project (University of California-Universidad de Madrid) and from the Universidad Complutense de Madrid under Multidisciplinary Project PR486/97-7477/97.

Partial results were presented at the ICO-19th Conference on ‘Optics for the quality of life’ (Florence, Italy, August 25-31, 2002)

References

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**Figure 1.** - One single fiber model. R: radius of each fiber. The wave-vector of the incoming radiation: $\beta$, is defined at the Z-direction (parallel to the optic axis of the waveguide). The magnetic field is plane polarised at the XY-plane.

**Figure 2.** - N coupled waveguides. $d$: average distance between two adjacent (parallel) fibers. The impact plane XY defines the plane where the amplitude of the light scattered by the fiber array is determined (see text for details).

**Figure 3.** - A model for mechanical-optical effect: An external force $F$ is applied and defined in some arbitrary XY plane. The conditions of incidence of light are similar as in Figs.1 and 2. Notice that in this work we are considering arrays for which the angle $\theta$ is small.

**Figure 4.** - Spatial dependence of the forward-scattering light intensity. Dotted-line curve: Zero applied force, $F_y=0$. Dashed-line curve: $F_y=0.01$ N, X–polarized light. solid-line curve: $F_y=0.01$ N, Y–polarized light.

**Figure 5.** - Forward-scattering intensity against the strength $F_y$ of the applied force. The solid-line curve corresponds to X-polarized light and the dotted-line, to Y-polarized light.