Devaluation Beliefs and Debt Crisis:
The Argentinian Case†

José-María Da-Rocha
Universidad Carlos III de Madrid
and Universidade de Vigo

Eduardo L. Giménez
Universidade de Vigo

Francisco-Xavier Lores
Universidade de Vigo

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Abstract
Throughout the year 2001 the Argentine real GDP fell by 20 percent and the Investment Rate decreased by more than 20 percent of the GDP. After rejecting plans to reduce the fiscal deficit, the government made several announcements on changes in exchange rate policy in order to assist the recovery of the economy. At same time, the Trade Balance produced a huge surplus and the Argentine External Debt over GDP ratio increased so much that it forced the Argentinian government to default and, afterwards, devalue the peso 40 percent. We explore the relationship between default and the expectations on devaluation. We consider a simple small open economy where the government issues public debt denominated in foreign good and consumers can invest in domestic capital or in foreign securities also denominated in foreign good. We show that if consumers expect a devaluation, they reduce investments in domestic capital and increase foreign holdings. This reduces GDP and tax revenues and thus the government has an incentive to devalue in order to assist the process of recovery of the economy. But if a government devalues it requires a higher fraction of GDP to repay its external debt. That is, the government faces a trade off between recovering the economy and increasing the future cost of repaying the external debt. We find that depending on the level of debt and given an expectation of devaluation two types of crises can arise: if the level of debt is low the government devalues but does not default; for a higher level of debt, the government devalues and defaults to cancel the future cost of repaying the debt. We have calibrated our model to match the main features of the Argentine crisis and we show that the External Debt over GDP ratio was in a crisis zone where the government found it optimal to default and to devalue.

Keywords: Debt crisis, Devaluation, Argentina

JEL Classification: E6,F3,F34.
1 Introduction

Throughout the year 2001 the Argentinian GDP fell by more than 20 percent and investment decreased by more than 5 percent of GDP. At the same time, the trade balance yielded a surplus, and foreign reserves fell dramatically. The ratio of external debt to GDP increased so much that it forced the Argentinian government to default in December 2001. Afterwards, in January 2002, the government devalued the peso by 40 percent. Figure 1 documents these facts.

[Figure 1 about here.]

What happened during 2001 in Argentina? On March 16, President De La Rúa rejected the plan presented by Economics Minister López Murphy to reduce the fiscal deficit. The new minister, Domingo Cavallo presented a new economic plan in the lower house of the Argentinian congress. On March 28, the congress refused to allow Cavallo to cut government salaries and pension expenditure, and the government sold debt to cover the deficit. Between April and August, several announcements on changes in the exchange rate policy were made. First, on April 12, Cavallo announced that the peso would peg to the euro (and maybe to the yen). In May, the government announced economic plans that included currency changes, and, on June 18, the Argentine government announced a complex set of new economic policies, including the installation of multiple exchange rates to help the country’s exporters. In July, the Province of Buenos Aires announced the issue of a new currency to pay bills (the patacón) and in August, the “Banco de la Nación” limited sales of dollars to a one-to-one rate with the peso.

[Figure 2 about here.]

Figure 2 reports the daily series of Argentinian reserves. As the announcements on changes were made, the reserves fell. An IMF aid package in September led to a recovery of reserves. But, on October 30, the government could not sell new debt and started to restructure its debt, finally forcing pension funds to buy government bonds. On December 23, the government defaulted, and, on January 11, 2002, the government devalued the peso, after a week of suspended convertibility.
In this paper we will consider a simple small open economy with three assets - domestic capital, foreign securities and public debt - to study the government’s incentives to devalue and to repay or default on the debt. We will show that expectations on devaluation account for the default on debt.

Our theory is simple. As the expectations of devaluation increase, domestic agents modify their portfolio by reducing their investment in domestic capital and increasing their foreign asset holdings. This reduces GDP and tax revenues. We assume that once a government devalues, the expectations vanish and the economy recovers its past levels of investment and GDP. A government has an incentive to devalue so as to increase the future levels of output, consumption, and capital stock. However, if a government devalues, in the future it requires a higher fraction of GDP to repay its external debt (which is denominated in foreign good). In consequence, the government policy of devaluation faces a trade-off between recovering the economy, and increasing the future cost of repaying the debt.

Our main result shows that under a speculative attack the optimal government policy depends on its level of debt. If the level of debt is low, the government devalues to increase capital but does not default. For higher levels of debt, the government does not devalue and repays its debt because the cost of a default is higher than the benefits of a devaluation. Finally, for sufficiently high levels of debt, the government defaults, because repaying the debt is too costly, and devalues, once the default eliminates the future cost of repaying the debt. Our theory explains why we sometimes observe “good” devaluations, where the economy recovers or “bad” experiences where devaluations take place only after government default, and as a result the economy pays a severe productivity cost that reduces investment and output (as in Argentina in 2002).

We calibrate our model to match the main features of the 2001 Argentinian crisis: in particular we reproduce the investment rate, the government expending over GDP, the external debt over GDP in 2000 before the crisis and we match the reduction in investment rate, the increase in foreign holding, and the increase in trade balance surplus observed throughout 2001 in Argentina. We show that the level of Argentinian debt (45% over GDP) would be in a sure zone if the government had not made the announcements of changes in
the exchange rate regime: that is, if the expectation of devaluation had not existed. We also show that for the probability of devaluation consistent with the risk premium of the Argentinian Government bonds nominated in dollars issued in April 2001 the external debt of Argentina was in a crisis zone where the government found it optimal to default and to devalue.

The most closely related paper is that of Cole and Kehoe on the 1986 Mexico crisis. The similarities can be found in that the government cannot commit itself and behaves strategically. Furthermore, the government cannot use income taxes to reduce the external debt. The differences can be seen in that the source of panic in our model are the consumers who expected a devaluation in the following period. In this sense, our model shows that, when the external debt is denominated in foreign currency, speculative attacks based on announcement of change in the exchange rate policy can generate a debt crisis.

The paper proceeds as follows. In the next section we present the economic environment. Section 3 presents the definition of equilibrium. Section 4 characterizes the optimal behavior of private agents. Section 5 characterizes the levels of debt for which the government always devalues and never defaults and in section 6 we show that an expectation of devaluation with default can exist. Section 7 characterizes government behavior in a self-fulfilling crisis and section 8 provides a numerical exercise for the Argentinian case. Finally, in the last section we conclude.

2 The economic environment

There are three agents in the economy –domestic consumers, international bankers, and government– and three assets –domestic capital, $K$, foreign security, $A$, and public debt, $B$. Both $A$ and $B$ are denominated in foreign good and can be exchanged for domestic goods at the real exchange rate $e$.

The international bankers have perfectly elastic demand for government debt at price $q$ and perfectly elastic supply of the foreign security at price $1/r^*$. The government provides an amount of public good $g$, obtains revenue from income taxes, $\tau$, and issues public debt, $B'$. Finally, domestic consumers are the owners of the capital, $k$, and foreign holdings, $a$, and they inelastically supply a unit of labor, and derive utility from private consumption.
and the government good.

**The consumers**

There is a continuum with measure one of identical, infinitely lived consumers who consume, invest, and pay taxes. The individual’s utility function is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t))$$

where $c_t$ is private consumption and $g_t$ is government consumption. The assumption of risk neutrality of consumers greatly simplifies the modelling of consumer behavior as in Cole and Kehoe (1996). We assume that $0 < \beta < 1$ and that $v$ is continuously differentiable, strictly concave, and monotonically increasing. We also assume that $v(0) = -\infty$. The households’ income, from foreign security holdings from previous period $a$ and factor payment (labor and capital), is devoted to paying taxes, consuming domestically produced goods $c$, and investing in domestic capital $k'$ and in foreign securities $a'$. The consumer’s budget constraint is

$$c_t + k_{t+1} + e_t[a_{t+1} + \Phi(a_{t+1})] = (1 - \tau)\alpha(z_t)\theta(e_t, e_{t-1})f(k_t) + c_tr^*a_t$$

where $r^*$ is the international interest rate in foreign good, and $e \in \{e, \bar{e}\}$ is the real exchange rate (pesos per dollar). We normalize $e = 1$. Here $k_t$ is the consumer’s individual capital stock; $\alpha$ is a multiplicative productivity factor that depends on whether or not the government has ever defaulted, denoted by the indicatrix $z$, and $0 < \theta < 1$ is another productivity factor that depends on whether the government devalues or not; $\tau$, with $0 < \tau < 1$, is the constant proportional tax on domestic income; and $f$ is a continuously differentiable, concave, and monotonically increasing production function that satisfies $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. The consumer is endowed with $K_0$ units of capital and $a_0$ units of foreign security at period 0. Finally, there is also an investment cost on international securities, represented by an increasing, convex function $\Phi(a_{t+1})$, with $\Phi'(0) = 0$. The existence of this function allows us to find the optimal allocations of foreign security.\(^1\)

There are three important assumptions. First, we are assuming that there is a technology that transforms domestic goods into foreign goods. The rate of transformation is

\(^1\)This trick is a common one in the small open economy literature. Otherwise, under arbitrage, it turns out to be difficult to compute the amount of resources devoted to the foreign assets.
the real exchange rate, \( e_t \), and, in order to simplify the model, we assume that no changes in nominal prices of domestic goods are expected or reported, so a nominal devaluation is also a real devaluation: the government, in choosing \( e_t \), also changes the real terms of trade between the domestic good and the foreign good.\(^2\) Second, if the government decides to default, there is a permanent negative productivity shock, as in Cole and Kehoe (2000) and other literature on financial economics. Finally, in order to determine the optimal level of a devaluation we assume that the period in which the devaluation happens, the economy is affected by a transitory negative shock in productivity

\[
\theta(e_t, e_{t-1}) = \begin{cases} 
\theta & \text{if government devalues } e_t > e_{t-1} \\
1 & \text{if government not devalues } e_t = e_{t-1}
\end{cases}
\]

There are two formulations that would rationalize our assumption that productivity falls after the government devalues. One scenario is that firms must renegotiate contracts and, in the short term, firms cannot substitute foreign inputs. We could assume, for example, that there is a foreign produced intermediate good, which cannot be substituted, whose price increases after a devaluation.\(^3\) Another scenario which may rationalize our assumption is that after devaluation the government increases trade taxes, set different exchange rates for exports and imports, or establishes quotas on trade. In summary, the government increases distortions in the economy and reduces output.

**The international bankers**

There is a continuum with measure one of identical, infinitely lived international bankers. The individual banker is risk neutral and has the utility function

\[
E \sum_{t=0}^{\infty} \beta^t x_t
\]

where \( x_t \) is the banker's private consumption. Analogous to Cole and Kehoe (1996) the assumption of risk neutrality of bankers captures the idea that the domestic economy is

\(^2\)This is a reduced form of a model where consumption and investment are composed of tradeable and non-tradeable goods.

\(^3\)To see how we can incorporate this scenario into our model, denote by \( F(n, k) \) the production function for output as a function the price of the intermediate good \( n \) and capital \( k \), and by \( p \) the price of the intermediate good. Then, if we require that \( f(k) = \max_n F(n, k) - pn, \theta(e_t, e_{t-1}) f(k) = F(n^*, k) - p(e_t/e_{t-1})n^* \), where \( n^* = \arg \max F(n, k) - pn \) we can reinterpret our model as one in which trade disruptions induce a production loss.

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small compared to world financial markets. Each banker is endowed with \( \bar{x} \) units of the consumption good in each period and faces the budget constraint

\[
x_t + q_t B_{t+1} + r^* a_t \leq \bar{x} + z_t B_t + a_{t+1}
\]

where \( q_t \) is the price of one-period government bonds that pay \( B_{t+1} \) in period \( t+1 \) when \( z_{t+1} = 1 \), that is, the government decides to repay its debts, and 0 if the government decides not to repay, i.e., \( z_{t+1} = 0 \).

The government

There is a single government, which is benevolent in the sense that its objective is to maximize the welfare of the consumers. In every period, the government makes three decisions: (i) it chooses the level of government consumption, \( g_t \), financed by household income taxes and by a new borrowing level \( B_{t+1} \); (ii) it decides whether or not to default on its old debt, \( z_t \in \{0, 1\} \); (iii) it chooses the real exchange rate, \( e_t \). Its budget constraint is

\[
g_t = \tau \alpha(z_t) \theta(e_t, e_{t-1}) f(k_t) + e_t [q_t B_{t+1} - z_t B_t]
\]

The government decides to pay, \( z_t = 1 \), or to default public debt, \( z_t = 0 \), and whether it devalues \( e_t > e_{t-1} \) or not \( e_t = e_{t-1} \). As in Cole and Kehoe (1996, 2000), national productivity is affected by a default (i.e., \( \alpha(z = 0) < \alpha(z = 1) \)) and the government loses access to international borrowing and lending after default. Finally, the market clearing condition for the government debt is \( b_{t+1} = B_{t+1} \), and we also assume that \( k_0 = K_0 \) and \( b_0 = B_0 \).

In each period, the value of an exogenous variable \( \xi_t \) is realized. We show that we can construct equilibria where, if the level of government debt \( B_t \) is above some crucial level and \( \xi_t \) is above another crucial level, then consumers will anticipate a devaluation and reduce domestic investment. This creates a self-fulfilling debt crisis in the sense that the reduction in domestic investment changes the government incentive to honor its debt. The government chooses to default and then to devalue.

The timing

We assume that the timing of actions within each period is the following:

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1. The government sells debt.
2. The international bankers, taking the price of debt as given, choose to buy or not to buy the debt.
3. The government decides to default or not, and chooses the exchange rate and government consumption.
4. The exogenous variable, $\xi$, is realized.
5. Consumers choose consumption and investment on the domestic and the foreign securities.

One crucial feature of our model is the timing of the consumers’ decisions. Given that they observe the sunspot after government decision-making, the government is unable to foresee the effects of sunspot on the consumers’ decisions.

### 3 Equilibrium

As in Cole and Kehoe (2000), the government cannot commit itself either to honoring its debt obligations or to following a fixed borrowing and spending path. It also cannot commit itself to modifying or not the real exchange rate, $e$. We closely follow Cole and Kehoe’s recursive equilibrium definition in which there is no commitment and the agents choose their actions sequentially.

When an individual consumer acts, he knows the following: his individual capital $k$, and foreign assets holding $a$, the aggregate state $s = (B, K, A, a_1, e_{-1})$; the government’s supply of new debt $B'$; the price that bankers are willing to pay for this debt $q$; the government’s spending, $g$, and default and devaluation decisions, $z$ and $e$, respectively; and the sunspot $\xi$. We define the state of the individual consumer as $(k, a, s, B', g, z, e, \xi)$. We denote the government’s policy functions by $B'(s)$, $g(s, B', q)$, $z(s, B', q)$ and $e(s, B', q)$; the price function by $q(s, B')$; and by $K'(s, B', g, z, e, \xi)$, $A'(s, B', g, z, e, \xi)$ the functions that describe the evolution of the aggregate capital and foreign asset stocks, all yet to be defined. The representative consumer’s value function is defined by the functional equation
The three policy functions of the consumers are \( c(k, a, s, B', g, z, e, \xi) \), \( k'(k, a, s, B', g, z, e, \xi) \) and \( a'(k, a, s, B', g, z, e, \xi) \). Because consumers are also competitive, we need to distinguish between the individual decisions, \( k_{t+1} \) and \( a_{t+1} \), and the aggregate values, \( K_{t+1} \) and \( A_{t+1} \). In equilibrium, given that all consumers are identical, \( k_{t+1} = K_{t+1} \) and \( a_{t+1} = A_{t+1} \).

As explained, the production parameters satisfy \( \alpha(s, z) = 1 \) if the government has not defaulted in the past and has not defaulted this period (otherwise, \( \alpha(s, z) = \alpha \)), and \( \theta(s, e) = 1 \) if the government has not devalued in this period (otherwise \( \theta(s, e) < 1 \)).

When an individual banker chooses his new debt level, he knows his individual holdings of government debt \( b \), the aggregate state \( s \), and the government’s offering of new debt \( B' \). The state of an individual banker is defined as \( (b, s, B') \). The representative banker’s value function is defined by the functional equation

\[
V_b(b, s, A, B') = \max_{b'} x + \beta E[V_b(b', s', A', B'(s'))] \\
\text{s.t. } x + q(s, B')b' + r^*A \leq \bar{x} + z(s, B', q(s, B'))b + A' \\
x > 0, \quad q(s, B')b' \leq \bar{x} \\
s' = (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e)
\]

\(^4\text{Note that to define the equilibrium we write } \alpha \text{ and } \theta \text{ as functions of the state. In the next section, we return to our original notation.}\)
Bankers are relatively passive: if $\bar{x}$ is sufficiently large, they purchase the amount of bonds offered by the government as long as the price of these bonds satisfies

$$q(s, B') = \beta E z(s', B'(s'), q(s', B'(s'))),$$

and the assumption that they behave competitively guarantees that they sell the amount of foreign assets demanded by consumers if $r^* = 1/\beta$.

The only strategic agent in the model is the government that makes decisions at two points in time. At the beginning of the period, when the government chooses $B'$, the government’s state is simply the initial state $s$. Later, after it has observed the actions of the bankers, which are summarized in the price $q$, it will choose whether or not to devalue, $e$, and default, $z$, which in turn determines the level of government spending, $g$, and the levels of productivity, $\alpha$ and $\theta$. This choice is given by the policy functions $g(s, B', q)$, $e(s, B', q)$ and $z(s, B', q)$. In consequence, at the beginning of the period the government knows how the price that its debt will bring, $q(s, B')$, depends on this state and on the level of new borrowing. The government also knows what its own optimizing choices $g(s, B', q(s, B'))$, $e(s, B', q(s, B'))$ and $z(s, B', q(s, B'))$ will be later. The government also realizes that it can affect consumption, $c$, domestic investment $K'$, foreign holdings, $A'$, and the production parameters, $\alpha$ and $\theta$, through its choices. The government’s value function is defined by the functional equation

$$V_g(s) = \max_{B'} E \left\{ c(K, A, s, B', g, z, e, \xi) + v(g) + \beta V_g(s') \right\}$$

s.t. $g = g(s, B', q(s, B'))$

$z = z(s, B', q(s, B'))$

$e = e(s, B', q(s, B'))$

$s = (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e)$

We denote by $B'(s)$ the government’s debt policy. At a later moment, the government makes its decisions on default, $z$, and devaluation, $e$, which determines the level of $\alpha$ and $\theta$ and the level of government spending, $g$. Given $V_g(s)$, we define the policy functions $g(s, B', q)$, $e(s, B', q)$ and $z(s, B', q)$ as the solutions to the following problem

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\[
\max_{g, z, e} E \{ c(K, A, s, B', g, z, e, \xi) + v(g) + \beta V_g(s') \}
\]
\[
s.t \quad g - \tau \alpha(s, z)\theta(s, e) f(K) \leq e[qB' - zB]
\]
\[
z = 0 \text{ or } z = 1
\]
\[
g \geq 0
\]
\[
\theta(s, e) = \begin{cases} 
\theta(e, \bar{e}) & \text{if the government devalues} \\
1 & \text{if government does not devalue}
\end{cases}
\]
\[
s' = (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e)
\]

**Definition of an equilibrium.**

An equilibrium is a list of value functions \(V_c\), for the representative consumer, \(V_b\) for the representative banker, and \(V_g\) for the government; policy functions \(c\), \(k'\) and \(a'\) for the consumer, \(b'\) and \(a'\) for the banker, and \(B', g, z\) and \(e\) for the government; a price function \(q\) and an interest rate \(r^*\); and equations of motion for the aggregate capital stock \(K'\) and foreign asset stock \(A'\) such that the following conditions hold,

1. Given \(B', g, z, e\) and \(\xi\), \(V_c\) is the value function for the solution to the representative consumer’s problem, and \(c\), \(k'\) and \(a'\) are the maximizing choices.

2. Given \(B', A', q, \) and \(z\), \(V_b\) is the value function for the solution to the representative banker’s problem, and the value of \(B'\) chosen by the government solves the problem when \(b = B\).

3. Given \(q, c, K', A', g, z\) and \(e\), \(V_g\) is the value function for the solution to the government’s problem first problem, and \(B'\) is the maximizing choice. Furthermore, given \(c, K', A', V_g,\) and \(B'\), then \(g, z\) and \(e\) maximize the consumer welfare subject to the government’s budget constraint.

4. \(q(s, B') = \beta E z(s', B'(s'), q(s', B'(s'))),\) and \(r^* = 1/\beta\).

5. \(K'(s, B', g, z, e, \xi) = k'(K, A, s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi) = a'(K, A, s, B', g, z, e, \xi)\) and \(B'(s) = b'(B, s, B')\).
Finally, consumers and bankers know that the government solves its problem each period, and therefore understand that, under some circumstances, the government will choose to default and/or to devalue.

4 The optimal behavior of private agents

The bankers’ optimal behavior depends upon the expectations they have about the government’s future repayment decision $z'$. If bankers expect that $z' = 0$, then they are not willing to buy any debt unless the price is 0. If bankers expect that $z' = 1$, then they are willing to buy any amount of the government debt up to $\bar{x}$ at price $\beta$. If bankers expect default to occur with probability $\pi$ they are willing to purchase whatever amount of bonds the government offers up to $\bar{x}$ at price $q = \beta(1 - \pi)$.

The consumers’ optimal policy depends solely on what they expect the values of the productivity parameters $\alpha$ and $\theta$ will be next period. There are several cases.

**No expectations of devaluation.** We start first with the cases where consumers have no expectations of devaluation. Consumers believe that the government will not devalue in the next period ($\pi = 0$). Then the first-order conditions are

$$\beta(1 - \tau)\alpha(z')f''(k') = 1$$

$$\Phi(a') = 0$$

$$c + k' = (1 - \tau)\alpha(z')\theta(e,e) f(k) + er^*a$$

If devaluation has occurred in period $t$ and the government has already defaulted, it is optimal for them to set the capital stock for the next period to a level $k^d$ that satisfies

$$\beta(1 - \tau)\alpha(0)f'(k^d) = 1$$

to set the level of foreign holdings $a' = 0$, and to consume whatever output is left over

$$c^{dd}(K, a) = (1 - \tau)\alpha(0)\theta(e,e) f(K) + er^*a - k^d$$

their consumption after devaluation and default occur is

$$c^{nd}(k^d, 0) = (1 - \tau)\alpha(0)f(k^d) - k^d$$
If devaluation has occurred in period $t$ and if the government has not defaulted, it is optimal for them to set the capital stock for the next period to a level $k^n$ that satisfies

$$\beta(1 - \tau)\alpha(1)f'(k^n) = 1$$

to set the level of foreign holdings $a' = 0$, and to consume whatever output is left over

$$c^{dn}(K, a) = (1 - \tau)\alpha(1)\theta(\xi, \tau)f(K) + \tau r^*a - k^n$$

their consumption after devaluation and not default occur is

$$c_0^{dn}(k^n, 0) = (1 - \tau)\alpha(1)f(k^n) - k^n$$

If the government does not devalue and has not defaulted, it is optimal for consumers to set the capital stock for the next period to the level $k^n$, to set the level of foreign holdings $a' = 0$ and consume whatever is left over

$$c_0^{nn}(K, a) = (1 - \tau)\alpha(1)f(K) + r^*a - k^n$$

and their consumption thereafter is $c_0^{nn}(k^n, 0)$.

If the government does not devalue but has defaulted, it is optimal for consumers to set the capital stock for the next period to the level $k^d$, to set the level of foreign holdings $a' = 0$ and consume whatever is left over

$$c^{nd}(K, a) = (1 - \tau)\alpha(0)f(K) + r^*a - k^d$$

and their consumption thereafter is $c^{nd}(k^d, 0)$.

**Expectations of devaluation.** We are interested in studying the cases in which consumers believe that the productivity parameter $\theta$ will be equal to $\theta(\xi, \tau)$ for the next period because the government has not previously devaluated, but consumers believe that the government will devalue during the next period ($\pi = 1$). Then, the first-order conditions are

$$\beta(1 - \tau)\alpha(\xi')\theta(\xi, \tau)f'(k') = 1$$

$$1 + \Phi(a') = \frac{1}{\theta(\xi, \tau)}$$

$$c + k' = (1 - \tau)\alpha(\xi')f(k) + \left[r^*a - a' - \Phi(a')\right]$$
If the government does not devalue and has not defaulted it is optimal for consumers to set the capital stock for the next period to a level $k^{dn}$ that satisfies
\[
\beta(1 - \tau)\alpha(1)\theta(\xi, \bar{\pi})f'(k^{dn}) = 1
\]
to set the level of foreign holdings $a^{dn}$ that satisfies
\[
1 + \Phi'(a^{dn}) = \frac{\bar{e}}{e}
\]
and consume whatever is left over
\[
c_1^{an}(K, a) = (1 - \tau)\alpha(1)f(K) + [r^*a - a^{dn} - \Phi(a^{dn})] \xi - k^{dn}
\]

If consumers believe that the government will devalue the next period ($\pi = 1$) and the government does not devalue and has defaulted, it is optimal for them to set the capital stock for the next period to a level $k^{nd}$ that satisfies
\[
\beta(1 - \tau)\alpha(0)\theta(\xi, \bar{\pi})f'(k^{nd}) = 1
\]
to set the level of foreign holdings $a^{dn}$ and consume whatever is left over
\[
c_1^{nd}(K, a) = (1 - \tau)\alpha(0)f(K) + [r^*a - a^{dn} - \Phi(a^{dn})] \xi - k^{nd}
\]

5 Devaluation without Default

In this section we will show that an equilibrium exists in which the government brakes the expectations of devaluation imposing a cost to the economy in the current period and maintaining in the future a higher level of debt. For low levels of debt a threshold on government debt will exist under which the government prefers to bear the cost of a devaluation, so that the subsequent recovery of the productivity will permit it to distribute this cost throughout future periods. We will denote this level by $b$.

We suppose initially that the government always pays its debt\(^5\) and that the consumers believe that the government will devalue in the following period, i.e. $\pi = 1$. The bankers do not experience panic and always buy all the debt issued to the level $\bar{\pi}$ at the price $q = \beta$. We will compare the payments that the government obtains by devaluing and not devaluing

\(^5\)Later on we will show that this is so in equilibrium.
to find the level of debt $b$. The payment the government obtains after devaluing and not defaulting is

$$V^{dn}(s, B_0, B_1) = c^{dn}(K_0, a_0) + v\left(\tau\alpha(1)\theta(\epsilon, \bar{\epsilon})f(K_0) + \bar{\epsilon}(\beta B_1 - B_0)\right) + \frac{\beta}{1-\beta}\left\{c_0^{dn}(k^n, 0) + v\left(\tau\alpha(1)f(k^n) - \bar{\epsilon}(1 - \beta)B_1\right)\right\}$$  \hspace{1cm} (1)

while if not devaluing and not defaulting

$$V^{nn}(s, B_0, B_1) = c^{nn}(K_0, a_0) + v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) + \frac{\beta}{1-\beta}\left\{c_1^{nn}(k^{dn}, a^{dn}) + v\left(\tau\alpha(1)f(k^{dn}) - (1 - \beta)B_1\right)\right\}$$  \hspace{1cm} (2)

The threshold $b$ will be the higher level of debt $B_0$ that verifies

$$V^{dn}(s, B_0, B_1) \geq V^{nn}(s, B_0, B_1)$$  \hspace{1cm} (3)

That is to say, for low levels of debt, despite consumer expectations on devaluation, the government does not devalue and repays its debt.

To determine the level of debt $b$, however, it is necessary to characterize the conduct of the government relating to the new debt. It is optimal for the government to maintain a constant level of spending $g_{t+1} = g_t$ and, hence, of its debt. Both depend on initial conditions $(K_0, B_0)$.

If the government has chosen to devalue, given that it is constant, government consumption is given by

$$g^d(B_0, K_0) = \tau\alpha(1)\left[\beta f(k^n) + \theta(\epsilon, \bar{\epsilon})(1 - \beta)f(K_0)\right] - \bar{\epsilon}(1 - \beta)B_0$$  \hspace{1cm} (4)

while government debt stays constant at

$$B^d(B_0, K_0) = B_0 + \frac{\tau\alpha(1)}{\bar{\epsilon}}\left[f(k^n) - \theta(\epsilon, \bar{\epsilon})f(K_0)\right].$$  \hspace{1cm} (5)

In the case that the government does not devalue, the constant government consumption will be given by

$$g^n(B_0, K_0) = \tau\alpha(1)\left[\beta f(k^{dn}) + (1 - \beta)f(K_0)\right] - (1 - \beta)B_0$$  \hspace{1cm} (6)

while government debt stays constant at

$$B^n(B_0, K_0) = B_0 + \tau\alpha(1)\left[f(k^{dn}) - f(K_0)\right].$$  \hspace{1cm} (7)
Given initial conditions \((K_0, B_0)\), when government consumption is constant, the government’s payoff from devaluing and not devaluing (1) and (2) is given, respectively, by

\[
V^{dn}(s, B_0, B^d(B_0, K_0))
\]

and

\[
V^{nn}(B_0, B^n(B_0, K_0)).
\]

We now argue that when government expenditure is constant, and for \(\beta\) sufficiently high, there is a unique \(b^* > 0\) such that

\[
V^{dn}(s, b^*, B^d(b^*, K_0)) = V^{nn}(s, b^*, B^n(b^*, K_0))
\]

When the constraint \(V^{dn} \geq V^{nn}\) is violated, i.e. \(B_0 > b^*\), in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or it may choose to devalue with a non-stationary expenditure by issuing a new debt level \(B_1\), to be different from \(B^d(B_0, K_0)\), and then maintain this level thereafter. Let \(B_1(B_0, K_0, a_0)\) be the value of \(B_1\) that satisfies \(V^{dn}(B_0, B_1) = V^{nn}(B_0, B_1)\), if such value exists. If no such \(B_1\) exists, then it is optimal for the government not to devalue. We now present a characterization of the equilibrium.

**Proposition 1.** For \(\beta < 1\) sufficiently close to 1 and \(\bar{c}_t\) sufficiently high, there exists a continuous (and increasing) function \(b(K, a)\) and a positive debt level \(b^*\), such that the following results occur.

(i) If \(0 \leq B_0 \leq b^*\), then the economy converges to a stationary equilibrium with devaluation, no default and constant government expenditure

\[
g_1 = g_2 = g^d = \tau \alpha(1) \left[ \beta f(k^n) + \theta(1 - \beta) f(K_0) \right] - \bar{c}(1 - \beta) B_0
\]

and constant government bonds \(B_1 = B_2 = B^d = B_0 + \frac{\tau \alpha(1)}{\delta} \left[ f(k^n) - \theta f(K_0) \right].\)

(ii) If \(b^* \leq B_0 \leq b(K_0, a_0)\), then the economy converges to a stationary equilibrium with devaluation, no default and the dynamics for the government expenditure is \(g^d_1 < g^d < g^d_2\) and constant at this level thereafter, and for the government bonds \(B_0 < B_1\) and constant at this level thereafter.

(iii) If \(B_0 > b(K_0, a_0)\), then the outcome is not in the devaluation no default equilibrium.
The most interesting case is $K_0 \leq k^n$, where the government issues new debt before devaluing (Figure 3). The reason is that it tries to distribute the cost of the devaluation among every period and to smooth its expenditure. If $B_0$ is small the government does not have any limit to issue new debt to maintain the public expenditure constant. After recovery it can face the future higher payment of the debt with higher tax revenue. The highest level of debt for which it is possible to transfer the cost of the devaluation to the new debt and completely smooth the public expenditure is $b^*$.

If $B_0 > b^*$ is not possible to distribute all the cost of the devaluation over time by issuing new debt and therefore the government must transfer part of the cost to a reduction of the public expenditure of the current period. If it tried to maintain the public expenditure constant the cost of repaying the debt in the future would be so high that the government would prefer not to devalue.

Proposition 1 establishes that there exists a level of debt that equalizes the benefits of the devaluation with the cost that this devaluation causes. First, a devaluation increases the debt service in the future because as devaluation helps the recovery of the economy the government issues more debt to smooth the public expenditure. Moreover, a devaluation also means an increase in the future cost of repaying the debt. These two effects are collected by the term $e(1 - \beta)B_1$ in (1). Second, the devaluation makes consumer expectation disappear, so that investment will increase from $k^{dn}$ to $k^n$, with the consequent increase in consumption and income. Note that while benefits are independent of the level of debt, costs are increase within it. Figure 4 reflects this intuition.

6 Devaluation with Default

When the level of debt is very high the government has no incentive to repay its debt. With respect to the case where the level of debt was low, now the cost of productivity that provokes a default is not high enough to oblige the government to repay its debt. Besides, if the government decides not to repay its debt, it will also decide to devalue, since a future

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6Note that the closer $\beta$ is to one the relevant benefits and costs are those provided in the future.
cost of repaying the debt does not exist. Thus, expectations are eliminated and domestic investment recovers.

This section characterizes two critical thresholds of debt. The first one, denoted by \( b \), determines a zone where the government always repays and therefore never devalues. The second, denoted by \( B \) determines another zone where the government never pays and therefore always devalues.

The levels of debt between \( b \) and \( B \) constitute a crisis zone in which, if the consumers expect a devaluation, this always happens and the government does not repay its debt and devalues. On the contrary, if consumers do not panic the government will not devalue and repays its debt. Furthermore, we will show that the optimal response of the government consists of reducing its debt to escape from the crisis zone.

In order to show that a crisis zone can exist, we follow the strategy of Cole and Kehoe (2000) and show that: first, whenever the level of debt is lower than the critical level \( b \), even if consumers expected a devaluation and bankers decide not to buy the new debt (observing the lower level of domestic investment), the government will not devalue and will repay its debt; second, whenever the level of debt is higher than the other critical level \( B \), even if consumers do not expect a devaluation and bankers will buy all the debt issued, the government will default and will devalue. These two levels of debt determine three zones: (1) the no crisis zone, where the government does not devalue and repays its debt; (2) the crisis zone without expectations of devaluation, where the government always defaults and devalues; (3) the crisis zone, where if consumers believe that a devaluation will happen, the government defaults and then devalues.

**No Crisis Zone**

To see how the government does not devalue and repay its debt, we study the case where the government repays even if bankers do not buy government bonds and consumers expect a devaluation (\( \pi = 1 \)).

In this zone the level of debt satisfies two conditions. First, the government prefers not to default and not to devalue than to default and devalue. Second, the government prefers not to default and not to devalue than to devalue and repay. Finally, in order to show that this equilibrium exists, we must show that the level of debt satisfies two conditions: we must show that the upper threshold found in proposition 1 is lower than the threshold found in
this case.

We will characterize the maximum level of debt (\(\tilde{b}\)) for which the government prefers to repay and not devalue. We will assume that, given that consumers expect a devaluation, \(\pi = 1\), the bankers know that the government devalues only after defaulting on its old debt \(B\) despite new debt issued \(B'\) is offered to international bankers. Given this conjecture, the price of the new government bonds falls to \(q = 0\).

The payoff to the government after devaluing and defaulting is given by

\[
V^{dd}(s, B_0, B_1) = c^{dd}(K_0, a_0) + v\left(\tau\alpha(0)\theta(e, \bar{e})f(K_0) + \bar{e}\beta B_1\right) + \frac{\beta}{1 - \beta} \left[c^{nd}(k^d, 0) + v\left(\tau\alpha(0)f(k^d)\right)\right]
\]

with \(B_1 = 0\) because bankers do not buy any government bonds, i.e., \(V^{dd}(s, B_0, 0)\).

The two constraints on government debt, which must be satisfied simultaneously in any equilibrium with no default and no devaluation, are

\[
V^{dn}(s, B_0, 0) \leq V^{nn}_1(s, B_0, 0)
\]
\[
V^{dd}(s, B_0, 0) \leq V^{nn}_1(s, B_0, 0)
\]

Let us define

\[
(V^{dd} - V^{nn}_1)(B_0, B_1) = H^{dd-nn}_1 + v\left(\tau\alpha(0)\theta(e, \bar{e})f(K_0) + \bar{e}\beta B_1\right) - v\left(\tau\alpha(1)f(K_0) + \beta B_1 - B_0\right) + \frac{\beta}{1 - \beta} \left\{v\left(\tau\alpha(0)f(k^d)\right) - v\left(\tau\alpha(1)f(k^{dn}) - (1 - \beta)B_1\right)\right\}
\]

where \(H^{dd-nn}_1 \equiv c^{dd}(K_0, a_0) - c^{nn}_1(K_0, a_0) + \beta/(1 - \beta) \left[c^{nd}(k^d, 0) - c^{nn}_1(k^{dn}, a^{dn})\right]\).

Let \(\tilde{b}(K, a)\) be the largest value of \(B_0\) for which the government weakly prefers to repay its debt and not devalue, even if it cannot sell new bonds at a positive price, i.e., \((V^{dd} - V^{nn}_1)(\tilde{b}(K_0, a_0), 0) = 0\). We refer to the range of debt value for which both constraints are satisfied as the no crisis zone, \(B \in (\tilde{b}(K_0, a_0), \bar{b}(K_0, a_0)]\). The following proposition establishes when we can find a non-empty no crisis zone, and shows that in the equilibrium characterized in proposition 1 the government always pays its debt.

**Proposition 2.** For \(\beta < 1\) sufficiently close to 1 and \(\bar{e}\) sufficiently high, there exists a continuous (and increasing) function \(\tilde{b}(K, a) > b(K, a)\), such that there exists a non-empty interval of levels of government debt \(B\), \(b(K_0, a_0) < B < \tilde{b}(K_0, a_0)\) where the government will not devalue and repays its debt.
Crisis without expectations

In order to determine the zone where crisis with default can be possible, we show that there exists a level of debt for which, even if consumers have no expectations on devaluation \((\pi = 0)\) and the bankers buy government debt, the government will default and then devalue. This level of debt determines the zone where crisis can only occur if consumers believe that a devaluation will take place in the next period.

Let be \(\bar{B}(K_0, a_0)\) the lowest level of debt for which the government prefers to default and devalue rather than not default and not devalue when no expectations on devaluation exist. Formally this level of debt must satisfy

\[
V^{dd}(s, B_0, B_1) \geq V^{nn}_0(s, B_0, B_1)
\]

where \(V^{nn}_0\) is the government’s payoff of not devaluing given by

\[
V^{nn}_0(s, B_0, B_1) = c^{nn}_0(K_0, a_0) + v(\tau \alpha(1)f(K_0) + (\beta B_1 - B_0)) + \frac{\beta}{1 - \beta} \{c^{nn}_0(k^n, 0) + v(\tau \alpha(1)f(k^n) - (1 - \beta)B_1)\}
\] (9)

In consequence, for an intermediate level of government bonds, the government will devalue and default if consumers expect a one-probability devaluation \((\pi = 1)\), and repay and does not devalue if consumers believe that no devaluation will occur \(\pi = 0\). See Lemma 4 in the appendix for the characterization of the equilibrium for \(B > \bar{B}\).

**Proposition 3.** For \(\beta < 1\) sufficiently close to 1 and \(\bar{e}\) sufficiently high, there is a non-empty interval of levels of government debt \(\bar{B} < B \leq \bar{B}\), for which crises with devaluation are possible.

[Figure 5 about here.]

In summary, there exist four zones (see Figure 5): (1) If \(B \leq \bar{b}\), the government devalues but repays its debt; (2) If \(\bar{b} < B \leq \bar{b}\), the government does not devalue and repays its debt; (3) If \(\bar{b} < B \leq \bar{B}\), if consumers reduce investments, the government devalues and does not repay its debt; (4) If \(\bar{B} < B\), the government always defaults and devalues.

[Figure 6 about here.]
The cost of the default is a fraction $\alpha(0)$ of the gross national product and therefore independent of the level of debt, while the benefits of the default are increasing in the level of initial debt. Figure 6 represents this intuition. The level of debt $\bar{B}$ is the level of debt such that benefits and costs of a default are equal. It is important to realize that for levels of debt lower than $\bar{B}$ the government always repays its debt if the consumers do not expect a devaluation ($\pi = 0$), since, as opposed to Cole and Kehoe (2000), the panic is not felt by the international bankers.

Figure 7 shows us the conditions under which the four zones exist. A default eliminates the future cost of repaying the debt and always increases the benefits of a devaluation. The level of debt characterized by proposition 2, $\bar{b}$, is the level of debt for which the benefits of a devaluation with default are equal to the net cost of a default. For levels of debt higher than $\bar{b}$ and lower than $\bar{B}$ the government never defaults because the cost of a default is always higher than the benefits of a devaluation with default. Then the government always repays its debt and does not devalue. Moreover, for levels of debt higher than $\bar{b}$ and lower than $\bar{B}$ the benefits of a devaluation with default are greater than the net cost of the default. Then the government defaults and devalues, because the net benefits of the devaluation compensate the net cost of the default. Finally, note that the benefits of the devaluation depends on consumer expectation. If the consumers have not expectations on devaluation ($\pi = 0$), the benefits of the devaluation are zero and the government never defaults and devalues for levels of debt lower than $\bar{B}$. In summary, for levels of debt between $\bar{b}$ and $\bar{B}$ the equilibrium depends on the devaluation expectations of the consumers.

7 Self-Fulfilling Devaluation Crises

We can now characterize the optimal government behavior in equilibria in which devaluation can occur with a positive probability $0 < \pi < 1$ depending on realizations of the sunspot variable $\zeta$.

A self-fulfilling devaluation crisis arises when there are two possible equilibrium results, one in which the government does not devalue and chooses to repay the old debt, and another in which the government devalues and defaults on the existing debt. Self-fulfilling crises are
possible in these equilibria for certain values of the fundamentals \((K, B)\); the realization of the sunspot variable determines which of these two results ensues.

In equilibrium, if \(\zeta \leq \pi\) and \(B\) is greater than the crucial level \(\bar{b}(K, a)\), then consumers predict that the government will devalue. Consumers reduce their investment in domestic capital and increase their foreign asset holdings. This reduces output and tax revenues in the next period and bankers are therefore not willing to pay a positive price for the new debt offered and thus provoke a default. If, however, \(\zeta > \pi\), then consumers predict that the government will not devalue. Because \(\zeta\) is uniformly distributed on the unit interval, \(\pi\) is both the crucial value of \(\zeta\), and the probability that \(\zeta \leq \pi\). If \(B\) is less than or equal to the crucial level \(\bar{b}(K, a)\), however, then no crisis can occur, no matter what the realization of \(\zeta\). If \(\zeta \leq \pi\), a crisis takes place if the debt is above \(\bar{b}(K, a)\) and below the upper threshold, which we now denote \(\bar{B}(K, a, \pi)\) since this threshold will also vary with \(\pi\). In previous sections we have analyzed the limiting cases where \(\pi = 0\) and \(\pi = 1\).

Before characterizing government behavior in this equilibrium, we need to know for what regions of \((B, K)\) values a self-fulfilling devaluation crisis is possible and for what regions, devaluation and default are the only outcome.

The lower threshold \(\bar{b}(K, a)\) does not change. No crisis equilibrium is possible if the government weakly prefers to repay its debt, even if it cannot sell new bonds and consumers predict a devaluation. Explicit characterization of the upper threshold on debt \(\bar{B}(K, a, \pi)\) is more difficult here because, as we shall see, optimal government policy will not, in general, be stationary in the crisis zone. We can explicitly characterize the upper threshold on debt under a stationary debt policy where the capital stock is equal to \(k^n\). Let \(B^s(\pi)\) be the largest value of \(B\) for which
where we have denoted \( \hat{\beta} = \beta(1 - \pi) \). As \( \pi \) tends to 0 this constraint tends to \( V^{dd} - V_0^{nn}(B, B) \leq 0 \) in Lemma 4; hence \( B^s(0) = B^s \).

**Lemma 1.** If the economy is such that, in the \( \pi = 1 \) crisis equilibrium, the stationary upper threshold on debt implied by lemma 4 satisfies \( B^s(0) > \overline{b}(k^n) \) then for any probability \( \pi \) and for \( K_0 = k^n \), there is a non-empty region of debt levels \( \overline{b}(k^n, a) < B < \overline{B}(k^n, a, \pi) \).

We now construct an equilibrium in which devaluation and default occur with positive probability. Suppose that \( K_0 = k^n \) and \( B_0 > \overline{b}(k^n, a) \), and the government is faced with the following choices in period 0: devaluate and default now; plan to run the debt down to \( \overline{b}(k^n, a) \) or less in \( T \) periods if no devaluation occurs; or never run the debt down. For each of these choices, we can calculate the expected payoff. The equilibrium is determined by the choice that yields the maximum expected payoff. Assuming that \( B_0 \leq B^s(\pi) \), the government maintains a constant level of government spending if a devaluation does not occur but is possible. If the government plans to run its debt down to \( \overline{b}(k^n, a) \) in \( T \) periods, we can use the government’s budget constraints to calculate that level of government spending:

\[
g^n_T(B_0) = \tau \alpha(1)f(k^n) + \frac{\hat{\beta}T - 1}{1 - \hat{\beta}T}B_0 - \frac{1 - \hat{\beta}}{1 - \hat{\beta}T}B_0 \]

If the government chooses never to run its debt down to \( \overline{b}(k^n, a) \), then government spending is

\[
g^\infty(B_0) = \tau \alpha(1)f(k^n) - (1 - \hat{\beta})B_0 \]

We can now calculate the expected payoff of running the debt down to \( \overline{b}(k^n, a) \) in \( T \).
Lemma 2. For any $\hat{\gamma}$ and $T$, the government’s payoff when its policy is to lower its debt to $B_0$, $\pi$, $k^n$, and the price function on government debt are given by

$$V^T(B_0) = c_0(k^n, a) + v\left( g^T(B_0) \right) + \frac{1}{\hat{\beta}} \left\{ (1 - \pi) \left[ c_0(k^n, 0) + v\left( g^T(B_0) \right) \right] + \pi \right\}$$

$$+ \pi c_1(k^n, 0) + \pi v\left( g^T(B_0) \right) + \pi V_\pi$$

where $\hat{\beta} = \beta(1 - \pi)$ and

$$V_\pi = \beta \left\{ c_0(k^n, a) + v\left( \tau \alpha(0) f(k^n) \right) \right\} + \beta^2 \left\{ c_0(k^n, 0) + v\left( \tau \alpha(0) f(k^n) \right) \right\} / (1 - \beta)$$

To determine $T$, we merely choose the maximum of

$$V^1(B_0), V^2(B_0), \ldots, V^\infty(B_0)$$

where

$$V^\infty(B_0) = c_0(k^n, a) + v\left( g^T(B_0) \right) + \frac{1}{\hat{\beta}} \left\{ (1 - \pi) \left[ c_0(k^n, 0) + v\left( g^\infty(B_0) \right) \right] + \pi \right\}$$

$$+ \pi c_1(k^n, 0) + \pi v\left( g^T(B_0) \right) + \pi V_\pi$$

Lemma 2. For any $K_0$ and $B_0 \leq B^*(\pi) - \tau \alpha(1)(f(K_0) - f(k^n))$, if we denote by $V^T$ the government’s payoff when its policy is to lower its debt to $B(k^n)$ in $T$ periods while keeping $\alpha$ constant, then a $T \in \{1, 2, \ldots, \infty\}$ that maximizes $\{V^1(B_0), V^2(B_0), \ldots, V^\infty(B_0)\}$ exists, and the following are true:

(i) If $K_0 \geq k^n$, as $B_0$ increases, $T(B_0)$ passes through critical points where it increases by one period. Furthermore, for $\pi$ close enough to 0, there necessarily are regions of $B_0 \leq B^*(\pi)$ with the full range of possibilities $T(B_0) = 1, 2, \ldots, \infty$;

(ii) If $K_0 < k^n$, then the debt may increase in the first period, but afterwards follows the same characterization as in (i) since $K_1 = k^n$ and $B_1 \leq B^*(\pi)$.

We are now ready to characterize crisis equilibria

Lemma 3. For any $\pi > 0$ for which there exists a non-empty crisis zone $\overline{B}(k^n, a) < B \leq \overline{B}(k^n, a, \pi)$, there can exist a crisis equilibrium in which the transition function for capital and the price function on government debt are given by

$$K(B') = \begin{cases} 
  k^n & \text{if } B' \leq \overline{B}(k^n, a, \pi) \text{ and } \alpha = \alpha(1) \xi > \pi \\
  k^{dn} & \text{if } B' \leq \overline{B}(k^n, a, \pi) \text{ and } \alpha = \alpha(1) \xi < \pi \\
  k^d & \text{otherwise}
\end{cases}$$
\[
q(B') = \begin{cases}
  \beta & \text{if } B' \leq \bar{b}(k^n, a) \text{ and } z(s, B', \beta) = 1 \\
  \hat{\beta} & \text{if } \bar{b}(k^n) < B' \leq \bar{B}(k^n, a, \pi) \text{ and } z(s, B', \hat{\beta}) = 1 \\
  0 & \text{otherwise}
\end{cases}
\]

and, depending on \( B_0 \), the following results occur

(i) If \( K_0 \geq k^n \) and \( B_0 \leq \bar{b}(k^n, a) \) then \( c_0 = c^n(K_0, a_0) \) and all other equilibrium variables are stationary: \( K = k^n, a = 0, c_t = c^n(k^n, 0) \) for \( t \geq 1 \), \( B = B_0 - \tau \alpha(1) (f(K_0) - f(k^n)) \), \( g = \tau \alpha(1) f(k^n) - (1 - \beta)B \), \( q = \beta \) and \( e = \bar{e} \). In this case no devaluation occurs;

(ii) If \( \bar{b}(k^n, a) < B_0 \leq \bar{B}(k^n, a, \pi) \), then a devaluation and default occurs with probability \( \pi \) in the first period and every subsequent period in which \( B \) is greater than \( \bar{b}(k^n, a) \). If \( B_0 \leq B^*(\pi) - \tau \alpha(1) (f(K_0) - f(k^n)) \), optimal government policy involves running down the debt to \( \bar{b}(k^n, a) \) in \( T(B_0) \) periods, while smoothing government expenditures as described in Proposition 6. If \( T(B_0) \) is finite and a crisis does not occur, then following period \( T(B_0) \), the equilibrium outcomes are those in (i). For \( B_0 > B^*(\pi) - \tau \alpha(1) (f(K_0) - f(k^n)) \), the equilibrium converges to the outcome described in Lemma 2 in at most two periods.

(iii) If \( K_0 < k^n \) and \( B_0 \leq \bar{b}(k^n, a) \), then there is no possibility of a devaluation in period 0, and from period 1 onward, the outcomes correspond to those described in (i) if under the government’s optimal policy, \( B_1 \leq \bar{b}(k^n, a) \) or in (ii) if not.

(iv) If \( B_0 > \bar{B}(K_0, a, \pi) \), then the only outcome is the devaluation default outcome in which \( c_0 = c^{dd}(K_0, a_0) \), \( g_0 = \tau \alpha(0) \theta(\bar{e}, \bar{e}) f(K_0) \), and all other equilibrium variables are stationary: \( K = k^d, c = c^{dd}(k^d, 0), B = 0, a = 0, g = \tau \alpha(0) f(k^d), q = 0, \) and \( e = \bar{e} \).

8 A Numerical Exercise

This section presents a numerical exercise whose parameters have been chosen so that the initial period matches the situation of Argentina in 2000. We use the model to help us interpret events in Argentina in 2001. We show that the crisis zone for our stylized model of Argentina is fairly large, and that the evolution of the variables of the model matches the evolution of the aggregate variables of Argentina’s economy during 2001.
The utility function for the consumers and the government is
\[ E \sum_{t=0}^{\infty} \beta^t (c_t + \log(g_t)) \]

The technology and the feasibility constraint are given by
\[ f(K) = AK^s \]
\[ c + g + k' - (1 - \delta)k \leq AK^s + [qB' - B - a' - \Phi(a') + r^*a]e \]

and the adjustment cost function is given by
\[ \Phi(a) = \phi_1 + \frac{(\phi_2a)^2}{2} \]

The capital share in GDP was taken from Kydland and Zarakaga (2002), \( s = 0.4 \). The discount factor \( \beta = 0.963 \) corresponds to an international interest rate of 3.84% that was taken from the interest rate in 2001 of US. 1 year Government Securities Treasury bills. The permanent drop of the productivity associated with a default is taken from Cole and Kehoe (2000) and implies a fall in productivity of 5%, \( \alpha(0) = 0.95 \). The temporary drop of the productivity related to a devaluation is established to reproduce the reduction in the investment rate observed between the year 2000 and 2001, \( \theta(\xi, \bar{\epsilon}) = 0.9892 \) that represents a fall in productivity of 1.92%.

Setting the probability of devaluation \( \pi = 0.0473 \) the yield of the Argentinian Government bonds nominated in dollars with a year of maturity is \( 0.09 = [\beta(1 - \pi)]^{-1} - 1 \) that corresponds with the government bonds issued with those characteristics on April 19, 2001. This means a risk premium of a 5.16% upon the Argentinian government bonds. The previous exchange rate to the crisis is fixed in \( \xi = 1 \) and the exchange rate after the devaluation in \( \bar{\epsilon} = 1.4 \) that corresponds to the exchange rate set by the Argentine government on January 11, 2002. Table 1 shows the values of the parameters calibrated without solving the model.

The next six parameters \( \phi_1, \phi_2, \tau, A, \delta \) and \( a_0 \) are calibrated solving the model. The parameters \( \phi_1 \) and \( \phi_2 \) of the adjustment cost is fixed to reproduce the the investment rate in the Argentinian GDP 2000, \( i/y = 0.18 \), and the reduction in international reserves of the
Central Bank that during the year 2001 reached 9200 million of dollars, 3.42% of the 2001 output. The tax rate and the TFP, $A$, are calibrated from the steady state budget constraint of the government to reproduce the shares of government spending and public debt in the Argentinian GDP 2000: $g/y = 0.19$ and $B/y = 0.45$ respectively. We obtain a depreciation rate of $\delta = 0.0815$ for a capita-output ratio of $K/Y = 3$. Finally, the initial value of the foreign assets $a_0$ is chosen to reproduce the share in GDP of the trade balance in 2000, that is to say, a surplus of 0.41%. Table 2 shows the values of the parameters.

[Table 1 about here.]

[Table 2 about here.]

With these values of the parameters and with $K_0 = k^n$ the levels of debt that determine the different zones of the model are presented in Table 3. Consequently, the initial values of debt for which a self-fulfilling devaluation crisis can occur in the first period are between 19.59% and 236.78% of the output. The level of Argentinian debt over GDP in 2000 reached 45% of the output, which means that it was in the crisis zone. That means that given a reasonable value for the expectations on devaluation in Argentina in 2001, a low ratio in debt over GDP of 45% far below the maximum level of sustainable debt in the case of no expectation existing, 236.78%, was enough to induce the government to default and devalue.

[Table 3 about here.]

9 Conclusions

We build a model of a small open economy, when the external debt is denominated in foreign goods, to study the relationship between the announcements of change in exchange rate policy and the government incentive to default or repay its debt. We find that once the announcements of the devaluation were made the government incentive to default changes. The government defaults for levels of debt that, if the expectations of devaluation did not exist, would always repay. That is, when the external debt is issued in foreign currency, announcements of change in the exchange rate policy can generate a debt crisis.

The model is calibrated for the 2001 Argentinian crisis. We find that for the external debt over GDP ratio for the year 2000 (45% of GDP) and given expectations of devaluation
consistent with the risk premium of the Argentinian Government Bonds issued in April 2001, the Argentinian government was in a crisis zone where it found it optimal to default and to devalue.

The main policy implications of our paper is that in countries where the government cannot commit itself to honor its debt and peg to foreign currency a higher fiscal surplus is needed to avoid debt crisis when speculative attacks may occur. This is similar to Cole and Kehoe’s conclusions on the 1986 Mexican crisis.

Moreover, our paper also shows that the exchange rate is not a political instrument which can be used to manage structural problems and reduce the fiscal deficits of the government, especially when the external debt is denominated in foreign currency. Exchange rate is a price and consumers will behave according to expectations of its future level. If policy makers announce change in the exchange rate to “improve” the competitiveness of the economy in order to postpone fiscal adjustment plans, the speculative attack over the local currency induced by the domestic consumer can induce debt crisis for levels of external debt that otherwise will never be defaulted.
References


A.1 Proof of Proposition 1

Proof. We will define the following difference as

\[(V^{dn} - V_1^{nn})(B_0, B_1) = H_1^{nn} + v(\tau \alpha(1)\theta(\xi, \pi)f(K_0) + \pi(\beta B_1 - B_0)) - v(\tau \alpha(1)f(K_0) + (\beta B_1 - B_0))\]

\[+ \frac{\beta}{1 - \beta} \left\{ v(\tau \alpha(1)f(k^n) - (1 - \beta)\pi B_1) - v(\tau \alpha(1)f(k^{nn}) - (1 - \beta)B_1) \right\} \]

where \(H_1^{nn} \equiv c^{dn}(K_0, a_0) - c^{nn}_1(K_0, a_0) + \beta/(1 - \beta) [c^{nd}(k^d, 0) - c^{nn}_1(k^{nn}, a^{nn})].\)

Notice that \((V^{dn} - V_1^{nn})(0, B_1) > 0\) as \(\beta \to 1\), which requires \(c^{nn}_1(k^{nn}, a^{nn}) > 0\). \(K_0 = B_0\) and, hence, of its debt. Both depend on initial conditions \((K_0, B_0)\).

We argue that when the government expenditure is constant, and for \(\beta\) sufficiently close to one, there is a unique \(b^* > 0\) such that

\[V^{dn}\left(s, b^*, B^d(b^*, K_0)\right) = V_1^{nn}\left(s, b^*, B^n(b^*, K_0)\right)\]

This condition is guaranteed if the adjustment cost is sufficiently high so that \(k^{dn} - (r^* - 1)a^{dn} + \phi(a^{dn}) - (1 - \tau)\alpha(1)[f(k^n) - f(k^{dn})] > k^n\) which implies \(c^{nn}_1(k^n, 0) > c^{nn}_1(k^{nn}, a^{nn})\), and also \(g^d_1 > g^n_1\).
let us write this constraint as \((V^{dn} - V_1^{nn})(b^*) = 0\) where

\[
(V^{dn} - V_1^{nn})(b^*) = H^{dn-nn}_1 + v\left(\tau\alpha(1)\theta(\xi, \tau)f(K_0) + \tilde{\epsilon}(\beta B^d(b^*, K_0) - b^*) \right) - v\left(\tau\alpha(1)f(k^n) - (1 - \beta)\tilde{\epsilon}B^d(b^*, K_0) \right) - v\left(\tau\alpha(1)f(k^n) - (1 - \beta)\tilde{\epsilon}B^d(b^*, K_0) \right) - v\left(\tau\alpha(1)f(k^n) - (1 - \beta)\tilde{\epsilon}B^d(b^*, K_0) \right)
\]

where \(H^{dn-nn}_1 = c^{dn}_0(K_0, a_0) - c^{nn}_1(K_0, a_0) + \beta \left[ c^{nn}_0(k^n) - c^{nn}_0(k^d, a^d) \right] / (1 - \beta).

Notice that \((V^{dn} - V_1^{nn})(0) > 0\) as \(\beta \to 1\), and that \((V^{dn} - V_1^{nn})(b) \to -\infty\) as \(b \to \tau\alpha(1)/\tilde{\epsilon} [\theta(\xi, \tau)f(K_0) + \beta f(k^n)/(1 - \beta)] \equiv B_0^{dn-nn}(B^n(b, K_0), \beta), \) i.e., \(g^d(b^*, K_0)\) goes to zero. Finally differentiating \(V^{dn} - V_1^{nn}\) yields

\[
\frac{d}{db} \left( V^{dn} - V_1^{nn} \right) < 0
\]

when \(\tilde{\epsilon}\) is sufficiently high.

Consequently, since \((V^{dn} - V_1^{nn})\) is continuous in \(b\), there is a unique \(b^*\) such that \(0 < b < B_1^{dn-nn}(B^n(b, K_0), \beta)\). That is, \((V^{dn} - V_1^{nn})(b^*) = 0\) and \((V^{dn} - V_1^{nn})(b) > 0\) for all \(B < b^*\), while \((V^{dn} - V_1^{nn})(b) < 0\) for all \(B > b^*\).

Whenever the constraint \(V^{dn} \geq V_1^{nn}\) is violated, i.e. \(B_0 > b^*\), in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or it may choose to devalue with a non-stationary expenditure by issuing a new debt level \(B_1\), to be different from \(B^d(B_0, K_0)\), and then maintain this level thereafter.

Let \(B_1(B_0, K_0, a_0)\) be the value of \(B_1\) that satisfies \(V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)\), if such value exists. If no such \(B_1\) exists, then it is optimal for the government not to devalue.

We now argue that there is a continuous increasing function \(b(K, a)\) such that for all \(b^* \leq B_0 \leq b(K_0, a_0)\) it is optimal for the government to devalue in period 0 and maintain a constant level of government expenditure different from period 1 on. In this case the government maintains a level of debt that differs from \(B_0\). For all \(B_0 > b(K_0, a_0)\), it is optimal for the government not to devalue. We then let

\[
b(K_0, a_0) = \max \left[ B_0(B_1, K_0, a_0) \right.
\]

subject to

\[
0 \leq B_1 \leq B_1(B_0, K_0, a_0)
\]

the constraint \(B_1 \leq B_1(B_0, K_0, a_0)\) binds if and only if the constraint \(V^{dn} \geq V_1^{nn}\) binds in period 0 when \(B_0 = b(K_0, a_0)\), i.e. \(V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)\). Differentiating \((V^{dn} - V_1^{nn})(B_0, B_1)\), we obtain

\[
\frac{\partial}{\partial B_0} \left( V^{dn} - V_1^{nn} \right) < 0
\]

when \(\tilde{\epsilon}\) is sufficiently high. Furthermore, since \((V^{dn} - V_1^{nn})(0, B_1) > 0\) as \(\beta \to 1\) and \((V^{dn} - V_1^{nn})(B_0, B_1) \to -\infty\) as \(B_0 \to B_0^{dn-nn}\), then there is a unique \(B_0(B_1)\) for which the constraint holds with equality; due to \(\partial(V^{dn} - V_1^{nn})(B_0, B_1)/\partial B_0 \neq 0\) the implicit function theorem implies that \(B_0(B_1, K_0, a_0)\) is continuous. Since \(B_0(B_1, K_0, a_0)\) is continuous in \(B_1\), it achieves a maximum on the compact constraint set.
The dynamics of the government expenditure and government bonds are the following,

\[ g_1^d = \tau \alpha(1) f(K_0) + \bar{e} \beta B_1 - \bar{e} B_0 \]
\[ g_2^d = \tau \alpha(1) f(k^a) - \bar{e}(1 - \beta) B_1 \]

In order to prove part (ii) recall first that for \( \bar{e} \) sufficiently high \( \partial(V^d - V^{nn})(B_0, B_1) \) < 0. Also observe that for \( \bar{e} \) sufficiently high, if \( g_2^d - g_1^d > 0 \) then \( \partial(V^d - V^{nn})(B_0, B_1) > 0 \), and if \( g_2^d - g_1^d < 0 \) then \( \partial(V^d - V^{nn})(B_0, B_1) < 0 \). In consequence, the implicit function theorem implies \( \frac{dB_2}{dB_0} \) a positive sign for the former, and a negative for the latter.

Second, observe that as \( B_0 \) increases up to \( B^* \), a positive sign is resulting in \( (V^d - V^{nn})(B_0, B_1) \) with constant public expenditure as indicated in part (i). However, as \( B_0 \) sets beyond \( b^* \) this positivity does not hold any longer, so an equilibrium with non constant public expenditure may exist. Having reached the threshold \( B_0 = b^* \) and then \( B^d = B^{d*} = b^* + \frac{\tau \alpha(1)}{\bar{e}}[f(k^a) - \theta f(K_0)] \), the dynamics of the public expenditure is, then, given by

\[ g_2^d - g_1^d = \tau \alpha(1) \left( f(k^a) - \theta f(K_0) \right) + \bar{e}(B_0 - B_1) = \bar{e} \left( (B_0 - b^*) - (B_1 - B^{d*}) \right) \] (10)

Now, beyond \( b^* \) the first increment is always positive. So the public expenditure can only be increased in this case by increasing government issue \( B_1 \), and lower than the difference \( B_0 - b^* \). Note that no other case is possible. Think of a decrease in public expenditure \( g_2^d - g_1^d < 0 \). As we indicated above, the implicit function theorem implies \( \frac{dB_2}{dB_0} < 0 \). If initially \( B_0 = b^* \) and afterwards it were increased, then \( B_1 \) would be consequently increased \( B_1 > B^{d*} \). This is a contradiction since (10) implies a positive increase in the public expenditure.

Finally it is easy to show that \( g_1^d < g^d < g_2^d \); since \( g_1^d - g^d = -\bar{e} \beta \left( (B_0 - b^*) - (B_1 - B^{d*}) \right) \)
and \( g_2^d - g^d = \bar{e}(1 - \beta) \left( (B_0 - b^*) - (B_1 - B^{d*}) \right) \).

A.2 Proof of Proposition 2

Proof. The proof follows the following steps.

First, we define the difference \( (V^d - V^{nn})(B_0, B_1) \) and \( (V^{dd} - V^{nn})(B_0, B_1) \), as well as some of their properties in the case where consumers have no expectations on devaluation. Two thresholds spring from these properties. Firstly, from proposition 1, the government, after deciding not to devalue, will devalue if the initial government bond level \( B_0 \) is lower than \( b(K_0, a_0) \), below which there will be no devaluation. Next, after the bankers decide not to buy government bonds, the government will default and devalue if the initial government bond level \( B_0 \) is higher than \( b(K_0, a_0) \), below which there will be no devaluation and no default.

Second, in order that the one-probability zone exists, we will show that it will be required that the threshold \( b(K_0, a_0) \) is lower than \( b(K_0, a_0) \), and then, a sufficiently high \( \beta < 1 \) can be found such that the zone exists.

Consider the definition and properties of \( (V^{dd} - V_{1}^{nn})(B_0, B_1) \) stated in proposition 1. That is, in proposition 1, for some \( \bar{e} \) high enough, it was proved that given any \( B_1 \), the difference \( (V^d - V_{1}^{nn})(B_0, B_1) \) is a decreasing function in \( B_0 \), i.e. \( \partial(V^d - V_{1}^{nn})(B_0, B_1)/\partial B_0 < 0 \), and that there exists a threshold \( b(K_0, a_0) \) such that \( (V^d - V_{1}^{nn})(b(K_0, a_0), B_1(b(K_0, a_0))) = 0 \) and for \( B_0 > b(K_0, a_0) \), \( V^d(B_0, B_1) < V_{1}^{nn}(B_0, B_1) \) for all \( B_1 \).

In addition, let us define now

\[ (V^{dd} - V_{1}^{nn})(B_0, 0) = H_{1}^{dd-nn} + \nu \left( \tau \alpha(0) \theta(\bar{e}, \bar{\tau}) f(K_0) \right) - \nu \left( \tau \alpha(1) f(K_0) - B_0 \right) \]

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Observe that \((V^{dd} - V^{mn}) (0, 0) < 0\) as \(\beta \to 1\), which implies that \(c^{dd} (k^d, 0) - c^{mn} (k^m, a^m) + v\left(\tau \alpha (0) f(k^d)\right) - v\left(\tau \alpha (1) f(k^{mn})\right) < 0\); and that \((V^{dd} - V^{mn}) (B_0, 0) \to +\infty\) as \(B_0 \to \tau \alpha (1) f(K_0) \equiv B_0^{dd-mm} (0, \beta)\). Here, it is easy to prove that \(\partial (V^{dd} - V^{mn}) (B_0, 0) / \partial B_0 = v' (\tau \alpha (1) f(K_0) - B_0) > 0\) so that there exists a threshold \((V^{dd} - V^{mn}) (b(K_0, a_0, \beta), 0) = 0\).

Now, in order the no crisis no devaluation zone exists, we must required that

\[
\bar{b}(K_0, a_0) < b(K_0, a_0, \beta). \tag{11}
\]

See Figure 1. First, considering \(B_1 = 0\) this condition holds since \(B_0^{dn-mm} (0, \beta) < B_0^{dd-mm} (\beta)\), due to \(\theta (\xi, \bar{\tau}) / \bar{\tau} < 1\). Now increasing \(B_1\), observe that \(B_0^{dn-mm} (B_1, \beta)\) increases (and eventually exceeds \(B_0^{dd-mm} (0, \beta)\)). However, in order that the condition (11) holds, we will require that \(^8 (V^{dn} - V^{mn}) (b_1 (K_0, a_0)) < 0\).

Under this assumption, for a \(\beta < 1\) sufficiently high, there exists some level \(B\) such that \(b(K_0, a_0) < B < B_0^{dd-mm}\). In addition, \(b(K_0, a_0, \beta)\) is strictly increasing in \(\beta\) and we can set \(\bar{b}(K_0, a_0, \beta)\) as close as \(B_0^{dd-mm}\) as wished. This means that the no crises zone without devaluation exists.

[Figure 9 about here.]

\[
\Box
\]

### A.3 Proof of Proposition 3

**Proof.** The proof consists in showing that the region exists for all \(B_1\).

In the region of crises, \(\pi = 1\), whatever the bankers do. It can be proved now that the difference \((V^{dd} - V^{mn}) (B_0, B_1)\) is increasing in \(B_1\) for a sufficiently high \(\bar{\tau}\).

In proposition 2 it was proved that in the crisis region \((V^{dn} - V^{mn}) (B_0, B_1) < 0\) when \((V^{dd} - V^{mn}) (B_0, B_1) > 0\).

Hence gains of default and devaluation for the government increase with the bonds bought by the bankers.

In addition, in the crises region the government prefers to devalue and default to default and not devalue for all \(B_1\), i.e., \((V^{dd} - V^{nd}) (B_0, B_1) > 0\), with

\[
(V^{dd} - V^{nd}) (B_0, B_1) = H_1^{dd-nd} + v\left(\tau \alpha (0) f(K_0) + \beta B_1\right) - v\left(\tau \alpha (0) \theta (\xi, \bar{\tau}) f(K_0) + \beta \bar{\tau} B_1\right)
\]

where \(H_1^{dd-nd} \equiv c^{dd} (K_0, a_0) - c^{nd} (K_0, a_0) + \beta (1 - \beta) \left[ c^{nd} (k^d, 0) - c^{dd} (k^m, a^m) \right], c^{dd} (K, a) = (1 - \tau) \alpha (0) f(K_0) + \left(\tau^* a - a^m - \phi (a^m)\right) \xi - k^{nd}\) and \(k^{nd}\) satisfies \(\beta (1 - \tau) \alpha (0) \theta (\xi, \bar{\tau}) f'(k^{nd}) = 1\). \(\Box\)

### A.4 Lemma 1

**Proof.** If the government prefers not to devalue and not default to devalue and to default, conditioned on keeping a constant debt level, then it certainly does so under the optimal debt policy; hence, \(B^* (\pi) \leq \overline{B} (k^n, a, \pi)\). As \(\pi\) increases, we can use the implicit function theorem to show that

\[^8\text{Observe that this is the same as to require } (V^{dd} - V^{mn}) (b(K_0, a_0), 0) < 0.\]

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\( B^*(\pi) \) decreases, making it more difficult for a non-empty interval \( \bar{b}(k^n, a) < B \leq B^*(\pi) \) to exist. Notice that \( B^*(0) > \bar{b}(k^n, a) \) implies that, if \( K_0 = k^n \) and \( B_0 = B_1 = \bar{b}(k^n, a) \), then the constraint 
\[ V^{dd} - V_0^{nn}(B, B) \leq 0 \]
with \( q = \beta \) and \( K_1 = k^n \) is strictly satisfied, and hence it is also satisfied by \( B_0 \) slightly greater than \( \bar{b}(k^n, a) \). Since this holds for any \( \pi \), \( \bar{B}(k^n, a, \pi) > \bar{b}(k^n, a) \).

\[ \text{A.5 Lemma 2} \]

Proof. Consider first the case where \( K_0 = k^n \). Either the maximum of \( V^T(B_0) \) is achieved for some finite \( T \) or it is not. If it is not, then

\[ V^T(B_0) = \lim_{T \to \infty} V^T(B_0) > V^T(B_0) \]

for all finite \( T \), and never running down the debt is optimal. We now argue that as \( B_0 \) increases above \( \bar{b}(k^n, a) \), it can pass through critical points where the optimal \( T \) increases by one period. For \( \bar{b}(k^n, a) < B_0 \leq \bar{b}(k^n, a)/(1 - \pi) \), it is optimal to set \( T = 1 \), because the yield from selling \( \bar{b}(k^n, a) \), \( \beta \bar{b}(k^n, a) \), is greater than \( \beta B_1 \) for any \( \bar{b}(k^n, a) < B_1 < B_0 \). As we increase \( B_0 \), we can pass through a critical point where the optimal \( T \) increases to \( T = 2 \). It cannot increase by more, because the optimal government policy is to steadily decrease \( B \) if it is to decrease at all. Therefore, there must be a region of values of \( B_0 \) where it is optimal to set \( T = 2 \) in between the regions where it is optimal to set \( T = 1 \) and where it is optimal to set \( T = 3 \). As we increase \( B_0 \) even further, we can increase \( T \), but we can never decrease it. To see why the optimal \( T \) can never decrease, observe that for \( \bar{b}(k^n, a) < B_0 < \bar{b}(k^n, a)/1 - \pi \),

\[ V^1(B_0) > V^2(B_0) > \cdots > V^\infty(B_0) \]

By setting \( T = 1 \) for \( B_0 \) in this region, the government can avoid crises without sacrificing government spending, as every period that it delays is costly. Differentiating our formula for \( V^T(B_0) \), we obtain

\[ \frac{\partial^2 V^T(B_0)}{\partial B_0 \partial T} = -v''(g^T(B_0)) \frac{\partial g^T(B_0)}{\partial T} = -v''(g^T(B_0)) \frac{\beta T-1 \ln \hat{\beta}(1-\hat{\beta})}{(1 - \hat{\beta}^T)^2} \left( \beta \bar{b}(k^n, a) - \hat{\beta} B_0 \right) < 0 \]

because \( B_0 > \bar{b}(k^n, a) \) and \( 0 < \hat{\beta} \leq \beta < 1 \). Hence

\[ 0 > \frac{d}{dB_0} V^\infty(B_0) > \cdots > \frac{d}{dB_0} V^2(B_0) > \frac{d}{dB_0} V^1(B_0) \]

Therefore, if \( V^T(B) \leq V^T(B_0) \) for \( T^* > T \), then for any \( B^* > B \), \( V^T(B^*) \leq V^T(B^*) \). Hence, if it is optimal to reduce the debt from \( B_0 \) to \( \bar{b}(k^n, a) \) in \( T \) periods, then for higher \( B_0 \), it cannot be optimal to reduce the debt in \( T - 1 \) periods.

As \( \pi \) tends to 0, the equilibria tend to those of the zero-probability crisis equilibria. For \( \pi \) close enough to 0, it is easy to show that there are necessarily regions of \( B_0 \) with the full range of possibilities \( T = 1, 2, \ldots, \infty \). Let \( V(B_0, T) = V^T(B_0) \) and \( g(B_0, T) = g^T(B_0) \) where we think of \( T \) as a continuous variable. We can differentiate our formula for \( V^T \) with respect to \( T \). If we can find that for fixed \( B_0 > \bar{b}(k^n, a) \) and for \( \pi \) small enough, \( \partial V(B_0, T)/\partial T > 0 \) for all \( T \), then we know that even for discrete \( T \), increasing \( T \) yields a higher expected payoff for the government.
Differentiating $V(B_0, T)$, we obtain

$$\frac{\partial V(B_0, T)}{\partial T} = \beta T^{-1} \left\{ \frac{\ln \beta (\beta b(k^n, a) - \beta B_0)}{1 - \beta T} \cdot \frac{V'}{g' (B_0)} + \frac{\beta \ln \beta (c_0^n(k^n, 0) + v (\tau \alpha(1) f(k^n) - (1 - \beta) b(k^n, a)))}{1 - \beta} - \frac{\beta \ln \beta ((1 - \pi) [c_0^n(k^n, 0) + v (g' (B_0))] + \pi c_1^n(k^n, 0) + \pi v [g' (B_0)] + \pi V''_{dd}}{1 - \beta} \right\} \text{ for all } T = \infty, \frac{\partial V(B_0, T)}{\partial T} \to 0, \text{ because } V(B_0, T) \to V^\infty(B_0). \text{ Even so, we are concerned with the sign of } \frac{\partial V(B_0, T)}{\partial T}. \text{ The benefit of increasing } T \text{ is that the government can maintain a higher level of government spending, and this benefit is captured by the first term in the formula above. Notice that as } \pi \text{ tends to 0, this benefit remains positive once we factor out } \beta T^{-1}. \text{ The cost of increasing } T \text{ is that the government risks crises for more periods, and this cost is captured by the last two terms in the formula above. Notice that as } \pi \text{ tends to 0, this cost goes to zero, even after we factor out } \beta T^{-1}.

Now fix a $B_0 > \bar{b}(k^n, a)$ and a $\pi$ for which $\partial V(B_0, T)/\partial T > 0$ for all $T$. The optimal government policy is to set spending equal to $g^\infty(B_0)$ and maintain debt at $B_0$. Our previous arguments now imply that for any $T$, there exists some initial $B$, $\bar{b}(k^n, a) < B < B_0$, for which the optimal government policy is to run its debt down to $\bar{b}(k^n, a)$ in $T$ periods. We know that for $\bar{b}(k^n, a) < B_0 < \bar{b}(k^n, a)/(1 - \pi)$, it is optimal to run down the debt in one period. We also know that for $B = B_0 > \bar{b}(k^n, a)/(1 - \pi)$, it is optimal to never run down the debt. Somewhere between $\bar{b}(k^n, a)/(1 - \pi)$ and $B_0$, all other intermediate possibilities must exist.

To rule out the possibility of there being a sudden jump from a finite $T$ being optimal to it being optimal to maintain the debt level constant, suppose to the contrary that such a jump does occur. Then, at the debt level $B$ where this jump occurs, we know that $V^T(B) = V^\infty(B)$, but $V^{T^*}(B) < V^T(B)$ for all $T^* > T$. Furthermore, $V^\infty(B^*) > V^T(B^*)$ for all $B^* > B$. The continuity of $V^T$ and $V^{T+1}$ implies that we can choose $\hat{B} > B$ so that $V^T(\hat{B}) > V^{T+1}(\hat{B})$. Since

$$0 > \frac{d}{dB} V^{T+1}(B) > \frac{d}{dB} V^T(B)$$

$V^T(B) > V^{T+1}(B)$ for all $B \leq \hat{B}$. Since $V^T(\hat{B}) \to V^\infty(\hat{B})$ as $T \to \infty$, however, we know that there exists a $\hat{T} > T + 1$ sufficiently great so that $V^\hat{T}(\hat{B}) > V^T(\hat{B})$. Consequently, if we restrict the government’s choices to the set $1, 2, \ldots, \hat{T}$ we know that at $B_0 = \bar{B}$ it would choose to run down its debt in $\hat{T}$ periods. Our previous arguments now imply that there has to be a region where $B < \bar{B}$ and where it is optimal to run down the debt in $T + 1$ periods, in particular where $V^{T+1}(\hat{B}) > V^T(\hat{B})$. This contradiction rules out the possibility of a sudden jump.

For the case when $K_0 \neq k^n$, a similar variational argument implies that under the optimal policy, $g$ is constant during the transition to $\bar{b}(k^n, a)$. Furthermore, if instead of $(K_0, B_0)$ as its state, the government has $(k^n, B_0 + \tau \alpha(1)(f(k^n) - f(K_0)), B_0 + \tau \alpha(1)(f(k^n) - f(K_0) < B^*(\pi))$, the government’s problem is unchanged, except that private consumption is different in period 0. Hence, the solution is unchanged.

\[\Box\]

### A.6 Proof Lemma 3

**Proof.** The characterization of the crisis equilibrium works similarly to that of the no devaluation equilibrium in Proposition 3. In the no devaluation equilibrium, the stationary debt policy char-
acterizes optimal government behavior and, implicitly, equilibrium results when the participation constraint does not bind. In the crisis equilibrium, $T(B_0)$ and $V^T_g(B_0)$ characterize optimal government behavior and, implicitly, equilibrium results when the participation constraint does not bind.

When the participation constraint does bind, we can use the identical logic as that in the proof of Lemma 4 to argue that, if $K = k^n$ then the equilibrium adjusts to that characterized by $T(B)$ and $V^T_g(B)$ in at most one period; in particular, if $B_1 > B^e(\pi)$, then $B_2 < B^e(\pi)$ and the government runs down its debt in $T(B_2)$ periods starting in the period after $K = k^n$. If $K_0 = k^n$, this is period 1, but if $K_0 \neq k^n$ and if the participation constraint binds in period 1, it is period 2. We also need to allow for the possibility that $K = k^n$ if the government needs to lower either $B_1$ or $B_2$ so much as to satisfy the participation constraints in period 0 or period 1 so that $B_1$ or $B_2$ is less than or equal to $\bar{b}(k^n, a)$. Otherwise, the proof follows the identical logic as that of Lemma 4. The notation involved in writing out the expressions for $V^d_g - V^n_g$ analogous to those found in the proofs in Lemma 4 and Proposition 3 is straightforward, but tedious. We omit it here.

A.7 Lemma 4

Lemma 4. For $\beta < 1$ sufficiently close to 1 and \( \bar{e} \) sufficiently high, there exists a continuous and increasing function $\bar{B}(K_0, a_0)$, and a positive debt level $B^*$, such that $\bar{B}(K_0, a_0) > B^*$ and $\bar{B}(K_0, a_0) > \bar{b}(K_0, a_0)$ for all $K_0, a_0$, such that the following occurs:

i) If $K_0 = k^n$, and $B_0 \leq B^*$, then the economy will be in the stationary no devaluation equilibrium in which government debt stays constant at its initial debt $B_0$.

ii) If $B_0 \leq \bar{B}(K_0, a_0)$, then the economy converges to the stationary no-default continuation equilibrium after at most two periods.

iii) If $B_0 > \bar{B}(K_0, a_0)$, then the outcome is that of devaluation and default equilibrium.

Proof. Observe that we are going to compare the constraints on the government’s debt that must be satisfied simultaneously in any equilibrium with no default and no devaluation, in the case that consumers have no expectations on devaluation and bankers buy government bonds

\[
V^{dd}(s, B_0, B_1) \geq V^{nd}(s, B_0, B_1) \\
V^{dn}(s, B_0, B_1) \leq V^{nn}(s, B_0, B_1)
\]

It is easy to show, for $\beta$ close to one and $\bar{e}$ sufficiently high, the two first constraints always hold. First, the government always devalues after a default even in the case that consumers do not expect a devaluation: given that,

\[
(V^{dd} - V^{nd}(B_0, B_1)) = c^{dd}(K_0, a_0) + v(\tau \alpha(0)(\bar{e}, \bar{e}) f(K_0) + \beta \bar{e} B_1) - c^{nd}_0(K_0, a_0) - v(\tau \alpha(0)f(K_0) + \beta B_1)
\]

this condition holds if $\bar{e}$ is sufficiently high. Second, if the government does not default it never devalues: for $\beta$ close to one and $\bar{e}$ sufficiently high, the following difference is always negative

\[
(V^{dn} - V^{nn}(B_0, B_1)) = H^{dn-nn} + v(\tau \alpha(1)(\bar{e}, \bar{e})f(K_0) + \bar{e}(\beta B_1 - B_0)) - v(\tau \alpha(1)f(K_0) + (\beta B_1 - B_0)) + \frac{\bar{e}}{1 - \beta} \left\{ v(\tau \alpha(1)f(k^n) - (1 - \beta)\bar{e} B_1) - v(\tau \alpha(1)f(k^n) - (1 - \beta)B_1) \right\}
\]

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where $H_{0}^{dd-nn} \equiv c_{0}^{dd}(K_{0}, a_{0}) - c_{0}^{nn}(K_{0}, a_{0})$.

The previous environment is so close to Cole and Kehoe (1999, Proposition 1) that i)-iii) can be followed in their proof. We have pointed out the slight difference shown in our model.

Finally, it remains to prove that the crises zone exists, i.e., $B(K_{0}, a_{0}) > b(K_{0}, a_{0})$. First, we define the following difference

\[
(V^{dd} - V_{0}^{nn})(B_{0}, B_{1}) = H_{0}^{dd-nn} + v(\tau \alpha(0) \theta(e, \bar{e}) f(K_{0}) + \beta \bar{e}B_{1}) - v(\tau \alpha(1) f(K_{0}) + (\beta B_{1} - B_{0}))
\]

\[
+ \frac{\beta}{1 - \beta} \left\{ v(\tau \alpha(0) f(k^{d})) - v(\tau \alpha(1) f(k^{n}) - (1 - \beta)B_{1}) \right\}
\]

where $H_{0}^{dd-nn} \equiv c_{0}^{dd}(K_{0}, a_{0}) - c_{0}^{nn}(K_{0}, a_{0}) + \beta/(1 - \beta) [c^{d}(k^{d}) - c^{n}(k^{n})]$.

Next, we will compare the constraints that set both levels $(V^{dd} - V_{1}^{nn})(B_{0}, 0)$ and $(V^{dd} - V_{0}^{nn})(B_{0}, B_{1})$. We will show that for $\beta < 1$ sufficiently close to one $(V^{dd} - V_{1}^{nn})(B_{0}, 0) > (V^{dd} - V_{0}^{nn})(B_{0}, B_{1})$ for all $B_{1}$. Given any $B_{1} > 0$, it is verified $(V^{dd} - V_{0}^{nn})(B_{0}, B_{1}) \to +\infty$ as $B_{0} = \tau \alpha(1) f(K_{0}) + \beta B_{1}$ is greater than $B_{dd-nn}$ stated in proposition 2, and in addition it is easy to show that $(V^{dd} - V_{1}^{nn})(B_{0}, 0) > (V^{dd} - V_{0}^{nn})(B_{0}, 0)$ given that subtracting both we find

\[
(V^{dd} - V_{0}^{nn})(B_{0}, 0) - (V^{dd} - V_{1}^{nn})(B_{0}, 0) = c_{1}^{nn}(K_{0}, a_{0}) - c_{0}^{nn}(K_{0}, a_{0}) + \\
\beta/(1 - \beta) \left[ c_{1}^{nn}(k^{nn}, a^{nn}) - c_{0}^{nn}(k^{n}, 0) + v(\tau \alpha(1) f(k^{nn})) - v(\tau \alpha(1) f(k^{n})) \right]
\]

which is negative for $\beta < 1$ sufficiently close to one. Then, by continuity of $(V^{dd} - V_{0}^{nn})(B_{0}, B_{1})$ this means that the one-probability crises zone exists. \qed
Table 1: Parameters Calibrated without Solving the Model

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>Parameter meaning</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 0.4$</td>
<td>Capital Share.</td>
<td>Kydland and Zarazaga (2000)</td>
</tr>
<tr>
<td>$\beta = 0.963$</td>
<td>Discount Factor.</td>
<td>1 year Gov. Securities Treasury bills</td>
</tr>
<tr>
<td>$\alpha(0) = 0.95$</td>
<td>Permanent drop in productivity.</td>
<td>Cole and Kehoe (2000)</td>
</tr>
<tr>
<td>$\theta(e, \bar{e}) = 0.9808$</td>
<td>Temporal drop in productivity</td>
<td>Investment rate reduction in 2001</td>
</tr>
<tr>
<td>$\pi = 0.0473$</td>
<td>Devaluation Probability</td>
<td>Risk premium of Argentinian Debt</td>
</tr>
<tr>
<td>$\varepsilon = 1$</td>
<td>Exchange rate pegged to US dollar</td>
<td></td>
</tr>
<tr>
<td>$\bar{e} = 1.4$</td>
<td>Exchange rate set on January 11, 2002</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Parameters Calibrated by Solving the Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibration Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1 = 71.54$</td>
<td>Adjustment cost</td>
</tr>
<tr>
<td>$\phi_2 = 4.3478 \times 10^{-5}$</td>
<td>Adjustment cost</td>
</tr>
<tr>
<td>$\tau = 0.2593$</td>
<td>Tax Rate</td>
</tr>
<tr>
<td>$A = 1206$</td>
<td>Scale factor</td>
</tr>
<tr>
<td>$\delta = 0.0815$</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$a_0 = 3436.9$</td>
<td>Initial foreign assets</td>
</tr>
<tr>
<td></td>
<td>Investment rate in 2000 ($i/y = 0.18$)</td>
</tr>
<tr>
<td></td>
<td>Reduction in reserves over GDP (3.42%)</td>
</tr>
<tr>
<td></td>
<td>Government spending over GDP in 2000 ($g/y = 0.19$)</td>
</tr>
<tr>
<td></td>
<td>External Debt over GDP in 2000 ($B/y = 0.45$)</td>
</tr>
<tr>
<td></td>
<td>Capital-Output ratio ($k/y = 3$)</td>
</tr>
<tr>
<td></td>
<td>Trade balance surplus (0.41%)</td>
</tr>
</tbody>
</table>
Table 3: Debt levels and percentage of output.

<table>
<thead>
<tr>
<th>Level</th>
<th>% of Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(k^n, a_0)$</td>
<td>24623</td>
</tr>
<tr>
<td>$\bar{b}(k^n, a_0)$</td>
<td>55672</td>
</tr>
<tr>
<td>$B^a(\pi)$</td>
<td>672930</td>
</tr>
</tbody>
</table>
1.1: Output.

1.2: Investment Rate.

1.3: Trade Balance over output.

1.4: Debt over output.

Figure 1: Argentinian facts I
Figure 2: Argentinian facts II
Figure 3: Bonds and government expenditure paths described at proposition 1 when $K_0 \leq k^n$. 
Figure 4: Devaluation without default.
\[ \pi = 0 \text{ no crisis} \]

\[ \pi = 1 \text{ crisis} \]

Devaluation

No Default

No Crisis Zone

Crisis Zone

Crisis without expectations

Figure 5: Existing zones I.
Figure 6: Default without expectations.
Figure 7: Existing zones II
Figure 8: Consumer decisions.
Figure 9: The no crises without devaluation zone upper threshold $\bar{b}(K_0, \beta)$, and the $(V^{dn} - V^{nn})(B_0, B_1)$ function evaluated at $B_1 = 0$. 