Quantum memory and all-optical switching in positive charged quantum dots via Zeeman coherent oscillations

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Abstract
Linear and nonlinear properties in an ensemble of single-hole spin InGaAs/GaAs quantum dots are theoretically studied. The application of a DC magnetic field in the semiconductor growth plane induces a Zeeman splitting of the hole levels which can be modeled like a four-level atom in the X-type configuration. All-optical and magneto-optical switching of a weak probe field are shown to be feasible in this system either by changing the Rabi frequency of an auxiliary control field or by changing the magnitude of the external magnetic field. We analyze the feasibility to store and release a weak pulse from the medium resulting from Zeeman coherent oscillations, thus the system is expected to be useful to develop a variable semiconductor optical buffer.

Keywords: coherent Zeeman oscillations, all-optical switch, slow light, dark-state polariton

1. Introduction
Photons can be considered as the most natural carriers of information, while atomic systems are expected to provide the best resources for their storage and manipulation. However it is extremely difficult to store photons in a medium for a long time and retrieve them on demand. In view of this, quantum states engineering and quantum information storage are key elements for the practical implementation of quantum information processing [1–5]. Recently, it has been shown both theoretically and experimentally that a light pulse, propagating through a medium composed of three-level atoms in the Λ configuration [6], suitably driven by another auxiliary field, can be stopped and later released in a controlled way (see the review in [7]). The process is interpreted in terms of inducing transient Raman coherence between two lower atomic states or in terms of an adiabatic evolution of the so-called dark-state polariton. Storage and retrieval of a weak probe pulse have been reported using EIT in cold atomic clouds [8], thermal gases [9] as well as in solid media [10]. These light storage schema are based on electromagnetically induced transparency (EIT) in ensembles of Λ-type atoms or multilevel schema involving various Λ-type subsystems. Examples of light storage in four-level systems such as double-Lambda, tripod [11] and inverted-Y atomic systems [12], have been discussed.

While most investigations have been performed in gaseous media, a transfer of the relevant techniques to the solid state is of considerable interest. Semiconductor nanostructures have been attracting a great deal of interest in the last decade in view of their intrinsic advantages concerning the bandwidth and their integrability with current devices. The manipulation of quantum coherence is a central issue in this field and is the key ingredient for the achievement of quantum information processing and quantum computation. The control of light propagation in a variety of semiconductor systems based on different physical mechanisms has been successfully demonstrated in experiments in the last decade and a half [13–15]. These include EIT [16–20] and optical storage in quantum-well (QW) structures [21], slow light by coherent population oscillations in semiconductor quantum amplifiers [22–25] and semiconductor quantum dots [26, 27], and slow light by exciton dephasing in semiconductor QWs [28]. In addition, voltage-controlled storage of light pulses in a coupled quantum dot nanostructure has been also predicted by Li et al [29, 30].

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The spin of an electron or a hole confined to a low-dimensional solid-state quantum system is a promising candidate for realizing a qubit, the fundamental building block of quantum based information processing devices. Recent optical studies on electron spin coherence in semiconductors have shown that spin coherence can be preserved over remarkably long times and length scales [31]. The robust electron spin coherence provides a promising platform for pursuing optical manipulation and control of quantum coherences in semiconductors, and for developing coherent semiconductor quantum devices [32]. However, interactions between the electron spin and its environment destroy the fragile coherence and lead to a loss of information [33–37], as found in experiments [38–41]. In order to mitigate the interaction with the nuclear spins, Brunner et al [42] have recently proposed the use of a hole spin from a p-type atomic wavefunction that conveniently goes to zero at the location of the nuclei, suppressing the hyperfine interaction.

In this paper, we present a theoretical model describing coherent properties of coupled hole spin in a stack of quantum dots and we analyze the possibility to use the system as a coherent optical memory. We will consider the so-called Voigt configuration in which the magnetic field is applied along the quantum-well plane. Generalization to an arbitrary field direction is straightforward. The Voigt case deserves special attention because it is the simplest case for experimental implementation and it provides an illuminating illustration of the underlying physics for the control of the quantum dot optical response. The Zeeman degeneracy of the atomic levels in the quantum dot is removed when a DC magnetic field is applied parallel to the growth plane, while light propagation remains along the growth direction, let us say the Z axis, in the so-called Voigt geometry. The control and probe fields co-propagate in the same waveguide. In this situation the electron and hole spins are quantized parallel to the applied magnetic field, and they are superpositions of the spin states quantized in the magnetic field direction \(|z, \pm\rangle = 1/\sqrt{2}[|x^+\rangle \pm |x^-\rangle]\), thus the spin vector oriented along the Z axis represents coherence between the spin eigenstates \(|x, \pm\rangle\) in the magnetic field. In the x basis, there are different electron and exciton spin eigenstates labeled as

\[
|\hat{1}\rangle = 1/\sqrt{2}[|1\rangle + |2\rangle], \quad |\hat{2}\rangle = 1/\sqrt{2}[|1\rangle - |2\rangle], \quad |\hat{3}\rangle = 1/\sqrt{2}[|3\rangle - |4\rangle], \quad \text{and} \quad |\hat{4}\rangle = 1/\sqrt{2}[|3\rangle + |4\rangle].
\]

The transition

\[
|\hat{1}\rangle \rightarrow |\hat{2}\rangle
\]
between the states $|1\rangle \leftrightarrow |4\rangle$ is mediated by right-handed circularly polarized light whereas the transition between the states $|2\rangle \leftrightarrow |3\rangle$ is allowed by left-handed circularly polarized light. Hence, the transitions $|1\rangle \leftrightarrow |4\rangle$, and $|2\rangle \leftrightarrow |3\rangle$ are mediated by linearly polarized light $\pi_x$, while transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ are mediated by linearly polarized light $\pi_y$ (see [45, 46]).

The DC magnetic field removes the degeneracy of the ground and excited states. The Zeeman separation of the electronic states in the QD ($2\Delta_c = g_e \mu_B B$), is different from the Zeeman separation of the ground levels ($2\Delta_g = g_g \mu_B B$). This non-equality originates from the difference in the Landé $g$-factors in these manifolds, which is typically in the order of $20 \mu$eV. In the case that the Zeeman splitting of the exciton states is larger than the linewidths, the quantum states of the hole can be described as a four-level system where $|1\rangle$ and $|2\rangle$ are the spin eigenvectors in the $x$ basis, split by the Zeeman effect. The upper levels $|3\rangle$ and $|4\rangle$ are the $X^+$ excitons consisting of two spin-paired holes and an unpaired electron with spin $\pm 1$, again in the $x$ basis. Typical values for the radiative decay rates are on the order of $\hbar \Gamma_3 = \hbar \Gamma_{32} = 0.5 \mu$eV at temperatures of 4.2 K [42]. Within this picture, a X-type system is established (see figure 1(b)). In what follows we restrict the process of light propagation to a single spatial dimension, let us say the $z$ axis. We consider a weak quantum field linearly polarized along the $Y$ axis with amplitude $E_p(z, t)$, and angular frequency $\omega_p$. This field couples the ground levels $|1\rangle$ and $|2\rangle$ with the two upper levels $|3\rangle$ and $|4\rangle$, respectively, and is given by

$$\hat{E}_p = \frac{1}{2} \sqrt{\frac{2\hbar \omega_p}{\varepsilon_0 V}} \hat{\sigma}_y E_p(z, t) e^{i \omega_p t} + \text{c.c.},$$

$\hat{\sigma}_y$ being the unitary polarization vector along the $Y$ axis. In addition, a classical control field with amplitude $E_c(z, t)$ and angular frequency $\omega_c$, propagates along the $Z$ axis and is assumed to be linearly polarized in the $X$ axis. This field couples the allowed transitions $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively. The control field is given by

$$\hat{E}_c = \frac{1}{2} \hat{\sigma}_y E_c(z, t) e^{-i \omega_c t} + \text{c.c.},$$

$\hat{\sigma}_y$ being the unitary polarization vector along the $X$ axis.

We use locally averaged atomic operators to perform the analysis of the light–matter interaction. We consider a medium of length $L$, which is divided into slices with a width $L_o$ over which the slowly varying field amplitudes exhibit negligible changes, containing $N \times V$ atoms, $V$ being the quantization volume. This volume is given by $V = AL_o$, $N$ and $A$ being the atomic density and the cross-sectional area of the pulses, respectively. We define the continuous atomic operators

$$\hat{\sigma}_{ab}(z, t) = \frac{1}{N} \sum_{j=1}^{N} \sigma_{ab}^j(z, t),$$

$\sigma_{ab}^j = |\alpha\rangle_j \langle \beta| (\alpha, \beta = 1, 2, 3, 4)$ being the quasi-spin operators. In the case that $\alpha \neq \beta$ these operators stand for the flip operator of the $j$th atom located at position $z_j$ from state $|\beta\rangle_j$ to state $|\alpha\rangle_j$, and $E_m(z, t) = \sum_k E_{mk}(t)e^{ikz}$. $E_m(t)$ ($m = x, y$) being the annihilation operator of the quantum fields. Using these continuous operators, the interaction Hamiltonian under the rotating wave approximation is given by

$$H = H_0 + H_E + H_C.$$  \hspace{1cm} (4)

The different terms in equation (4) are explicitly given by

$$H_0 = \hbar \sum_{m=1}^{4} \omega_m \hat{\sigma}_{mm},$$

$$H_E = -h g_3 E_p(z, t) e^{-i \omega_p t} \hat{\sigma}_{13} - h g_4 E_p(z, t) e^{-i \omega_p t} \hat{\sigma}_{24} + \text{H.c.},$$

$$H_C = -h \Omega_4(z, t) e^{-i \omega_c t} \hat{\sigma}_{31} - h \Omega_3(z, t) e^{-i \omega_c t} \hat{\sigma}_{32} + \text{H.c.},$$

(5) where H.c. stands for Hermitian conjugate. The dipolar moments are assumed to be real valued without loss of generality. $\Omega_3 = \mu_{32} \hat{E}_c/(2\hbar)$, and $\mu_{34} = \mu_{41} \hat{E}_c/(2\hbar)$ are the Rabi frequencies of the control field associated to transitions $|2\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$, respectively, and $g_j (j = 3, 4)$ stands for the coupling constant of the quantum field. In what follows we will consider that $\mu_{31} = \mu_{32}$, thus $\Omega_3 = \Omega_4 = \Omega_c$ and $g_3 = g_4 \equiv g$.

The time evolution of the system is governed by the set of Heisenberg–Langevin equations, which in an appropriate frame read as

$$\frac{\partial \hat{\sigma}_{14}}{\partial t} = -[\Gamma_{14} - i(\Delta_c \hat{\sigma}_{14}) + ig E_p e^{-i \omega_p t} \hat{\sigma}_{12} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{34} + i\Omega_c(\hat{\sigma}_{11} - \hat{\sigma}_{44}) + \hat{F}_{14}],$$

$$\frac{\partial \hat{\sigma}_{13}}{\partial t} = -[\Gamma_{13} - i(\Delta_c \hat{\sigma}_{13}) + ig E_p e^{-i \omega_p t} (\hat{\sigma}_{11} - \hat{\sigma}_{33}) - i\Omega_c(\hat{\sigma}_{11} - \hat{\sigma}_{44}) + \hat{F}_{13}],$$

$$\frac{\partial \hat{\sigma}_{12}}{\partial t} = -[\Gamma_{12} + i(\omega_c \hat{\sigma}_{12}) + ig E_p e^{i \omega_p t} \hat{\sigma}_{14} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{32} + i\Omega_c \hat{\sigma}_{13} - i\Omega_c \hat{\sigma}_{24} + \hat{F}_{12}],$$

$$\frac{\partial \hat{\sigma}_{34}}{\partial t} = -[\Gamma_{34} + i(2 \Delta_c \hat{\sigma}_{34}) + i\Omega_c \hat{\sigma}_{31} - i\Omega_c \hat{\sigma}_{24} + ig E_p e^{-i \omega_p t} \hat{\sigma}_{32} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{34} + \hat{F}_{34}],$$

$$\frac{\partial \hat{\sigma}_{24}}{\partial t} = -[\Gamma_{24} - i(\Delta_c + 2 \Delta_g \hat{\sigma}_{24}) + i\Omega_c \hat{\sigma}_{31} - i\Omega_c \hat{\sigma}_{24} + \hat{F}_{24}],$$

$$\frac{\partial \hat{\sigma}_{23}}{\partial t} = -[\Gamma_{23} - i(\Delta_c + 2 \Delta_g \hat{\sigma}_{23}) + ig E_p e^{-i \omega_p t} \hat{\sigma}_{21} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{33} + i\Omega_c(\hat{\sigma}_{22} - \hat{\sigma}_{33}) + \hat{F}_{23}],$$

$$\frac{\partial \hat{\sigma}_{32}}{\partial t} = -[\gamma_{41} + \gamma_{42} \hat{\sigma}_{44} + i\Omega_c \hat{\sigma}_{41} - i\Omega_c \hat{\sigma}_{14} + ig E_p e^{-i \omega_p t} \hat{\sigma}_{42} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{44} + \hat{F}_{44}],$$

$$\frac{\partial \hat{\sigma}_{31}}{\partial t} = -[\gamma_{31} + \gamma_{32} \hat{\sigma}_{33} + i\Omega_c \hat{\sigma}_{32} - i\Omega_c \hat{\sigma}_{23} + ig E_p e^{-i \omega_p t} \hat{\sigma}_{31} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{33} + \hat{F}_{33}],$$

$$\frac{\partial \hat{\sigma}_{22}}{\partial t} = \gamma_{23} \hat{\sigma}_{33} + \gamma_{24} \hat{\sigma}_{44} + i\Omega_c \hat{\sigma}_{23} - i\Omega_c \hat{\sigma}_{32} + ig E_p e^{-i \omega_p t} \hat{\sigma}_{24} - ig E_p e^{-i \omega_p t} \hat{\sigma}_{42} + \hat{F}_{22}. \hspace{1cm} (6)
Magnitudes \( \Delta_c = \omega_c - \omega_c \) and \( \delta = \omega_p - \omega_c \) stand for the detuning of the control field with regard to the \( \{|1\} \leftrightarrow |4\rangle \) transition, and the frequency difference of the probe field to the control field, respectively. In addition, \( \Gamma_{13} = \Gamma_{23} = (3 \omega_1 + 3 \omega_2)/2, \Gamma_{14} = \Gamma_{24} = (\omega_1 + 3 \omega_2)/2, \Gamma_{34} = (\omega_1 + 3 \omega_1 + \omega_2 + \omega_2 + \omega_2 + \omega_2)/2 \) denote the decay rate of the corresponding coherences and \( \gamma_{ij} \) stands for the spontaneous decay rates. We also assume the closing condition: \( \sum_{i=1}^4 \gamma_{ii} = \tilde{I}, \tilde{I} \) being the identity operator. The factors \( \tilde{F}_{ij} \) are \( \delta \)-correlated noise operators, i.e., \( \langle \tilde{F}_{ij}(t) \tilde{F}_{ji}(t') \rangle = D_{ij} \delta(t - t') \), where \( \delta \) stands for the Dirac distribution.

In addition, the evolution of the Heisenberg operator corresponding to the quantum field can be described in the slowly varying amplitude approximation by the Maxwell wave equation, namely

\[
\frac{\partial}{\partial t} E_p(z, t) + c \frac{\partial}{\partial z} E_p(z, t) = i Ng \left( \hat{\sigma}_{13} + \hat{\sigma}_{24} \right). \tag{7}
\]

Equations (6) can be written in a compact form as

\[
\frac{dR}{dt} + \Sigma = LR, \tag{8}
\]

\( L \) being a \( 15 \times 15 \) matrix of coefficients in equations (6) and \( \Sigma \) a (15, 1) vector. The Bloch vector is defined as 

\[
R(t) = \left[ \langle \hat{\sigma}_{13}(t) \rangle, \langle \hat{\sigma}_{31}(t) \rangle, \langle \hat{\sigma}_{24}(t) \rangle, \langle \hat{\sigma}_{42}(t) \rangle, \langle \hat{\sigma}_{41}(t) \rangle, \langle \hat{\sigma}_{14}(t) \rangle, \langle \hat{\sigma}_{41}(t) \rangle, \langle \hat{\sigma}_{32}(t) \rangle, \langle \hat{\sigma}_{22}(t) \rangle \right]^T, \tag{9}
\]

and super-index \( T \) stands for the transpose operation.

It is to be noted that some of the coefficients in the differential equations in equations (6) are time-dependent. In fact, there is no reference frame in which all the coefficients would become time independent. Therefore, to solve that system of equations we resort to use of the Floquet method by expanding the density matrix elements as Fourier series in terms of amplitudes that oscillate at the probe detuning and its harmonics. Matrix \( L \) and vector \( \Sigma \) are partitioned into terms with different time dependence and the expansion is limited up to first order in the probe field amplitude according to

\[
L = L_0 + \xi E_p e^{-i\delta t} L_1 + \xi E_p e^{i\delta t} L_{-1}, \tag{10}
\]

and

\[
\Sigma = \Sigma_0 + \xi E_p e^{-i\delta t} \Sigma_1 + \xi E_p e^{i\delta t} \Sigma_{-1}, \tag{11}
\]

where the expressions for each one of the matrices, \( L_0, L_1, \) and \( L_{-1} \), can be easily derived from equations (6). These expressions are substituted back into equation (8) and we arrive at the following equation

\[
\frac{dR}{dt} + \Sigma_0 + \xi E_p \Sigma_1 + \xi E_p \Sigma_{-1} = \left( L_0 + \xi E_p L_1 + \xi E_p L_{-1} \right) R. \tag{12}
\]

Using the Floquet theorem it is easy to find the stationary solution for \( R \) that will have only terms which oscillate at harmonics of the detuning \( \delta \). We assume that probe field amplitude \( E_p \) is weak enough so that the Floquet harmonic expansion can be truncated at third order,

\[
R = R_0 + e^{-i\delta t} g E_p R_1 + e^{i\delta t} g E_p^* R_{-1} + |g E_p|^2 H_0 + e^{-2i\delta t} (g E_p)^2 R_2 + e^{2i\delta t} (g E_p^*)^2 R_{-2} + |g E_p|^2 H_1 + e^{-i\delta t} (g E_p) |g E_p|^2 H_{-1} + e^{i\delta t} (g E_p^*) |g E_p|^2 H_{-1}. \tag{13}
\]

Equation (13) is substituted back into equation (12) and by equating the coefficients of the different harmonics of \( \delta \) corresponding to the same power in \( E_p \), we obtain the solutions for \( R_0, R_1, \ldots \) etc, which read as

\[
R_0 = L_0^{-1} \Sigma_0, \tag{14}
\]

\[
R_1 = -(L_0 + i\delta)^{-1} (\Sigma_1 + L_1 R_0), \tag{15}
\]

\[
R_{-1} = -(L_0 - i\delta)^{-1} (\Sigma_{-1} + L_{-1} R_0), \tag{16}
\]

\[
H_0 = -L_0 (L_1 R_{-1} + L_{-1} R_1), \tag{17}
\]

\[
H_1 = -(L_0 + i\delta)^{-1} (L_1 H_0 + L_{-1} R_2). \tag{18}
\]

The susceptibility experienced by the probe field is given by

\[
\chi(\delta) = \frac{N}{\epsilon_0} \mu_{31}(\langle \sigma_{13}^{(1)}(\delta) \rangle + \langle \sigma_{13}^{(3)}(\delta) \rangle)
+ \frac{N}{\epsilon_0} \mu_{42}(\langle \sigma_{24}^{(1)}(\delta) \rangle + \langle \sigma_{24}^{(3)}(\delta) \rangle), \tag{19}
\]

where the linear susceptibility terms \( \langle \sigma_{13}^{(1)}(\delta) \rangle \) and \( \langle \sigma_{24}^{(1)}(\delta) \rangle \) are obtained from \( R_1 \), whereas the nonlinear susceptibility terms \( \langle \sigma_{13}^{(3)}(\delta) \rangle \) and \( \langle \sigma_{24}^{(3)}(\delta) \rangle \) are obtained from \( H_1 \).

3. Linear and nonlinear properties of the X-type system

In view of the results obtained in section 2, we resort to analyzing both the linear and nonlinear properties of the current system. In the discussion which follows, we assume that Rabi frequencies, decay constants and optical detunings are scaled to \( \gamma_{41} = \gamma \). We also consider that the control field is tuned to resonance, i.e., \( \Delta_c = 0 \), unless otherwise stated, and that the lower level dephasing is negligible (\( \Gamma_{21} \approx 0 \)).

3.1. Absorption and dispersion of the probe field in the linear regime

Let us consider the case where the atomic levels are degenerated, i.e., \( \Delta_p = \Delta_c = 0 \) (the external magnetic field is turned off). Figure 2(a) depicts the imaginary part of the linear susceptibility \( \text{Im}[\chi^{(1)}(\delta)] \) experienced by the probe field versus the probe detuning \( \delta \), normalized to \( N|\mu_{31}|^2/(2\epsilon_0) \), for several values of the control field \( \Omega_0 \). In the case that \( \Omega_c < \gamma \), the spectrum of the probe field exhibits a perfect transparency window at the line center (\( \delta = 0 \)).

At first sight one would think that the dip in the absorption profile is similar to that produced in EIT. However in the present case the dip in the absorption originates from Zeeman coherent oscillations [47]. In fact, while in the EIT case an increase of the Rabi frequency of the control field merely shifts the Autler–Townes splitting, in the present situation the
Figure 2. Imaginary part of the linear susceptibility experienced by the probe field versus the probe field detuning $\delta$ normalized to $N|\mu_{31}|^2/(2\hbar\epsilon_0)$. (a) $\Omega_c = 0.1\gamma$ (solid line), $\Omega_c = 0.2\gamma$ (dashed–dotted line), and $\Omega_c = 0.75\gamma$ (dotted line). (b) $\Omega_c = 0.85\gamma$ (solid line), $\Omega_c = \gamma$ (dashed–dotted line), and $\Omega_c = 2\gamma$ (dotted line). (c)/(d) Real part of the linear susceptibility experienced by the probe field versus the probe field detuning corresponding to (a)/(b). The parameters used to produce the figure are: $\Delta_1 = 0$, $\Gamma_{21} = 0$, and $\gamma_{41} = \gamma_{42} = \gamma_{31} = \gamma_{32} \equiv \gamma$.

The qualitative behavior of the system changes dramatically when the control field is set to a value larger than the linewidth ($\Omega_c > \gamma$). This fact can be appreciated in figure 2(b), where the imaginary part of the linear susceptibility is depicted in this regime of Rabi frequencies of the control field, which resembles the behavior well-known of two-level atomic systems [48]. Transparency is maintained at resonance, but two gain sidelobes appear. This is a well-known feature caused by coherent population oscillations which, in the regime of high level excitations, dominate the Zeeman coherences. This behavior can be explained by considering the steady-state solution for the combination of coherences $\hat{\sigma}^{(1)}_1 = \hat{\sigma}^{(1)}_{14} + \hat{\sigma}^{(1)}_{23}$ (see appendix for more details), which can be shown to be given by

$$\hat{\sigma}^{(1)}_1(t \to \infty) = \frac{i}{[1 + i(\Delta_c + \delta)]} [g E_p(n_g^{(0)} - n_e^{(0)})$$
$$+ \Omega_c(\hat{\sigma}^{(1)}_{14} - \hat{\sigma}^{(1)}_{23})].$$

(20)

The inspection of equation (20) reveals that the component of the dipole moment oscillating at frequency $\delta$ has two contributions: the first one, that which is proportional to the probe field amplitude $E_p$, is equivalent to the term appearing in conventional theories of pump–probe spectroscopy. The second contribution originates from the Zeeman oscillations at the beating frequency $\delta$, is proportional to the Rabi frequency of the control field ($\Omega_c$), and is responsible for the qualitatively new features found in the numerical simulations.

In summary, the linear absorption of the probe field presents two different regimes of operation in the degenerate case. In the regime of weak control fields, transparency is achieved at the line center and is accompanied by a steep dispersion (see figure 2(c)). This extremely abrupt change of the dispersion can be used to slow down, and eventually stop, a probe pulse inside the medium, provided that the carrier frequency of the probe pulse coincides with the angular frequency of the control field and that its bandwidth does not exceed the width of the absorption profile. This fact opens up the possibility to use the current system as an optical memory and will be explored later in this work (see section 4).

In the regime of moderate control fields, the propagation of light signals changes from the subluminal to the superluminal regime (see figure 2(d)).
parameters are as in figure 2. \(\Omega_1\) \(\delta\) values of the Rabi frequency of the control field. The values \(\Omega_1\) \(\delta\) \(\Omega_1\) \(\delta\) of the linear susceptibility experienced by the probe field versus the probe field detuning.\(\) Figure 3.

In summary, a small change in the Rabi frequency \(\delta\) will become highly transparent for \(\delta = 0\), the medium, which is absorptive \(|\gamma_c|\). This will produce an interesting result: the control field acts as a knob for changing the level of absorption of the probe field. This behavior is of interest in order to obtain a semiconductor based all-optical switching device. To demonstrate such a possibility we present in figure 3(a) the imaginary part of the linear probe susceptibility versus the probe laser detuning \(\delta\). This produces the appearance of the \(\Lambda\)-type. In the case of negative detunings the system exhibits another minimum in the absorption which can be turned into gain once a certain Rabi frequency of the control field is reached. The residual displacements of the positive and negative minima that arise from the influence of the Rabi frequency of the control field can be understood by considering the process in the dressed state picture. In this regime of values for the external magnetic field, the four-level system behaves as nearly two independent three-level \(\Lambda\)-type systems, and each one of the subsystems lead to an EIT-like behavior when the corresponding two-photon resonance condition is achieved.

Now we focus our attention on the question of all-optical switching which originates from the change from high absorption state to a nearly transparent state when the applied magnetic field produces splitting of the levels less than the linewidth. In order to show the feasibility of this operation, we consider a weak probe field \(\Omega_p \ll \gamma\). We use the same parameters as in figure 3(a), whereas the control field is subject to a time variation of the form

\[
\Omega_c = 0.75 \left\{ 1 - 0.5 \sum_{j=0}^{2} \tanh[2(t - t_{2j+1})] \right\} + 0.5 \sum_{j=1}^{3} \tanh[2(t - t_{2j})], \tag{21}
\]

where \(t_1 = -14, t_j = t_1 + (j - 1)t_h,\) and \(t_h = 7 (j = 2, \ldots, 6),\) and time is specified in units of \(\gamma^{-1}\). We proceed to solve numerically the field equation for the probe field (7) together with the matter evolution equations (6). It is worth mentioning that the source term appearing in equation (7) is the full polarization, whereas we need to isolate the component of the polarization which oscillates at \(\delta\) to evaluate the numerical solution. In addition, the Floquet expansion is used in the matter equations (6) to isolate the terms oscillating at \(\delta\).

Figure 4(a) shows the time evolution of the probe field (solid line) when the control field \(\Omega_c\) is modulated as a train of tanh-like pulses as given in equation (21). The line...
corresponding to the time variation of the control field has been scaled to take similar values to the maximum of the output probe field for display purposes. When the control beam is off, a strong absorption takes place at $\delta = 0$ (see solid line in figure 3(a)), thus the amplitude of the output field is close to zero. However, when the control beam is turned on, the system becomes nearly transparent at the line center, in agreement with the dotted line in figure 3(a).

One way to measure the effectiveness of the switch is to compute the contrast $C$ of the output signal defined as

$$C = \frac{E_p(L)^{\text{on}} - E_p(L)^{\text{off}}}{E_p(L)^{\text{on}} + E_p(L)^{\text{off}}}.$$  \hfill (22)

$E_p(L)^{\text{on}} / E_p(L)^{\text{off}}$ being the output probe field when the control field is turned on/off. For the case considered in figure 4(a), a very high contrast value is obtained ($C = 0.9996$).

There is another possibility to obtain an optical switch for the probe field. This fact arises from the behavior of the linear susceptibility of the system: note that at the line center ($\delta = 0$), and in the absence of a magnetic field, the system exhibits transparency (see figure 2(a)). In addition, the system can lead to a strong absorption of the probe field (figure 3(a)) at the line center but for a non null magnetic field. In view of this fact, the temporal modulation of the external magnetic field allows the system to change from transparency to absorption, thus there still exists another possibility to produce switching in the system. This can be proved by considering the following variation of the magnetic field

$$B(t) = 0.09 \times 0.5 \left\{ \sum_{j=0}^{3} \tanh[2(t - t_{2j+1})] \right\} - \sum_{j=1}^{4} \tanh[2(t - t_{2j})],$$  \hfill (23)

where $t_1 = -14, t_j = t_1 + (j - 1)t_h$, and $t_h = 40$ ($j = 2, 8$), and time is also specified in units of $\gamma^{-1}$. The time variation described in equation (23) induces a change in the upper and lower level splittings which can shown to be given to a good approximation by

$$\Delta g = -0.00263B^2(t) + 1.21341B(t),$$

$$\Delta g = +0.00112B^2(t) + 2.54062B(t).$$  \hfill (24)

The coefficients appearing in the fittings in equations (24) are obtained from the data in figure 10 in [46] and are expressed in tesla. Matter and field equations are solved simultaneously and the results obtained are depicted in figure 4(b). The dotted line, corresponding to the time variation of the magnetic field, has been scaled to take a value close to the maximum of the output probe field for display purposes. Note that the timescale in this last simulation is larger than that used in figure 4(a), this indicates to us that the magneto-optical switch operates with a lower repetition rate than the all-optical switching. Finally, the contrast for this mode of operation reduces to $C = 0.9769$, which still represents a high value to distinguish between the on and off states of the switch.

3.2. Absorption and dispersion of the probe field in the nonlinear regime

We now focus on the third order susceptibility through numerical simulation. In comparison with conventional atomic systems, the light interaction in semiconductor quantum dots is strongly enhanced due to the existence of large dipole moments [43]. We assume realistic parameters for the system [42]: the radiative decay linewidth is taken on the order of 2.5 GHz, the control field is set to 0.1 $\gamma$ and the value for $\alpha_0$ is on the order of $10^3$. We will show that the current system can be engineered to produce a giant enhancement of the third order nonlinearity with vanishing linear absorption,
Figure 5. Imaginary part of the linear susceptibility $\chi^{(1)}$ (dashed line), imaginary/real part of the third order susceptibility $\chi^{(3)}$ (dashed–dotted/solid line). The control field is set to $\Omega_1 = 0.1 \gamma$ and is detuned from resonance by $\Delta_c = 0$ in (a), $\Delta_c = \gamma$ in (b) and $\Delta_c = 2.5 \gamma$ in (c). The Rabi frequency of the probe field is set to $\Omega_p = 0.001 \gamma$. The magnetic field used to produce the curves is $B = 0$ T, thus $\Delta_e = \Delta_g = 0$.

which is of interest in obtaining ultraslow optical solitons and nonlinear phase gates. To this end we present in figure 5 the imaginary part of the linear susceptibility (dashed line) and the real/imaginary part of the third order susceptibility (solid/dashed–dotted line) versus the detuning $\delta_p$ for different values of the detuning of the control field ($\Delta_c$) in the absence of an external magnetic field, which leads to a degenerate X-type system. The results obtained for the case $\Delta_c = 0$ are depicted in figure 5(a), where we can appreciate that at resonance we obtain transparency for the probe field, in accordance with figure 2(a). However, note that the real part of the nonlinear susceptibility, which is responsible for self-phase modulation, cancels out. Fortunately, this case varies with the tuning of the control field, which produces a non null $\text{Re}[\chi^{(3)}]$ with vanishing linear absorption, as shown in figure 5(b) for the case with $\Delta_c = \gamma$. The level of nonlinearity achieved can be modified by increasing the detuning: this will result in the controllability of the nonlinear absorption and nonlinear dispersion (see figure 5(c)). It is worth mentioning that the third order nonlinearity has been multiplied by a $10^4$ factor to have all the magnitudes on the same scale for display purposes.

4. Dark-state polaritons via Zeeman coherent oscillations

We now proceed to consider the question of how to store a pulse of light in a collection of quantum dots according to the description presented in section 2 and the results obtained in section 3 in relation to the linear susceptibility in the regime of moderate Rabi frequencies of the control field as depicted in figures 2(a) and (c). In doing this we will show that the transfer of the electromagnetic excitation onto a spin wave can be reversed by acting on the classical control field. We assume that the atomic levels are degenerated, and the Rabi frequency of the quantum field is weak enough that the number density of photons in the quantum field is much smaller than the atomic density ($N$). In such a case the atomic equations can be treated perturbatively in $E_p(z, t)$. In the discussion which follows we shall disregard the noise operators ($\hat{F}_{ij}(t) = 0$) since we will
consider the so-called adiabatic limit. We also consider the situation of equal decay rates (see [42]), as in the section 3. By keeping terms to first order in $E_p(z, t)$ in both equations (6) and in equation (7), we arrive at a reduced set of equations which describe the time evolution of the coherences and the quantum field of interest in the linear regime (see appendix for more details on the procedure), which in the case with $\delta = 0$ read

$$\frac{\partial \sigma_T^{(i)}}{\partial t} = -\Gamma_1 \sigma_T^{(i)} + i\Omega_T \left( \delta_T^{(i)} - \sigma_T^{(i)} \right) + i \eta \Omega_T E_p, \quad (25)$$

$$\frac{\partial \sigma_S^{(i)}}{\partial t} = -iG E_p \rho_S^{(0)w} + i\Omega_S \left( \delta_S^{(i)} - \sigma_S^{(i)} \right), \quad (26)$$

$$\frac{\partial \sigma_T^{(i)}}{\partial t} = -\left[ \Gamma_3 + \delta \right] \sigma_T^{(i)} - \frac{\partial \sigma_T^{(i)}}{\partial t}, \quad (27)$$

$$\frac{\partial \sigma_S^{(i)}}{\partial t} = -\Gamma_3 \sigma_S^{(i)} - i\Omega_S \left( \delta_S^{(i)} - \sigma_S^{(i)} \right), \quad (28)$$

$$\frac{\partial}{\partial t} E_p(z, t) + c \frac{\partial}{\partial z} E_p(z, t) = i Ng \sigma_T^{(i)}, \quad (29)$$

where the different magnitudes are defined as follows: $\sigma_T^{(i)} = \sigma_T^{(i)1} + \sigma_T^{(i)2}$, $\sigma_S^{(i)} = \sigma_S^{(i)1} + \sigma_S^{(i)2}$, $\delta_T^{(i)} = \delta_T^{(i)1} + \delta_T^{(i)2}$, $\delta_S^{(i)} = \delta_S^{(i)1} + \delta_S^{(i)2}$, $\sigma_T^{(i)} = \sigma_T^{(i)1} + \sigma_T^{(i)2}$, $\sigma_S^{(i)} = \sigma_S^{(i)1} + \sigma_S^{(i)2}$ and $n^{(0)} = n^{(0)}_g - n^{(0)}_c$. In the following discussion we assume that the different atomic operators are those with super-index (1), although we will omit it for simplicity in the notation.

To get a deep insight into the full process, it is adequate to separate the different phases of the process in the slowing down of the pulse (writing phase), its storage (storage phase), and the later release of the light pulse (reading phase). In doing this we follow the lines of [49, 50] with proper modifications in order to consider the present situation with X-type atoms. We have assumed the probe and the control fields to be tuned to resonance, i.e., $\delta = \Delta_c = 0$, while we allow the classical control field to be externally manipulated. Let us consider equations (25)–(28) while assuming that $\Omega_T \gg \gamma E_p$. It is easy to show that the following identity holds:

$$\sigma_T - \sigma_S = -\frac{i}{\Omega_T} \frac{\partial}{\partial t} \sigma_T. \quad (30)$$

The time evolution of magnitude $\sigma_T$ is easily derived from equation (30) together with equations (25) and (28), and it can be shown that it obeys the following second order differential equation:

$$\frac{\partial^2}{\partial t^2} \sigma_T + \Gamma_1 \frac{\partial}{\partial t} \sigma_T + 2\Omega_T^2 \sigma_T + g \Omega_T n^{(0)} E_p = 0. \quad (31)$$

During the writing phase, we can ignore the time derivatives appearing in the left-hand side of equation (31), thus producing the following value for the magnitude

$$\sigma_T^W(t_w) = -\frac{g n^{(0)}}{2 \Omega_T^2} E_p^W, \quad (32)$$

where $t_w$ stands for the time at which the writing phase ends. Note that super-index $W$ has been added to the magnitudes $E_p$, $\sigma_T$, and $\Omega_T$ to indicate that their values are those produced during this phase of the process. The approximation made to derive equation (32) will hold provided that the time duration of the light pulse, $T$, satisfies the following inequalities:

$$T \gg \frac{\gamma}{\Omega_T^W}, \quad \text{and} \quad T^2 \gg 1 \frac{\gamma^2}{\Omega_T^W}. \quad (33)$$

Provided that equation (33) holds, it becomes clear that the magnitude $\sigma_T$ can adiabatically follow the probe field, i.e. magnitude $\sigma_T$ contains a replica of the input field $E_p$, as indicated by equation (32).

It is easy to demonstrate that the source term in equation (29) can be expressed in terms of $\sigma_T$ and $\partial_t \sigma_T$ as follows

$$\frac{\partial}{\partial t} E_p(z, t) + c \frac{\partial}{\partial z} E_p(z, t) = i Ng \sigma_T^{(i)} \approx i Ng \left( -\frac{i}{\Omega_T} \frac{\partial}{\partial t} \sigma_T \right) \frac{\partial}{\partial t} E_p(z, t) = 0. \quad (35)$$

In view of the previous considerations, the probe pulse propagates as a traveling wave without attenuation, with a velocity $v_p$, i.e. $E_p^R(z, t) = E_p^W(0, t - z/v_p)$. The external manipulation of the control field $\Omega_T$ results in a reduction of group velocity of the light pulse inside the medium, as occurs in a $\Delta$-type atomic medium, allowing a complete map of the pulse which is stored in the Zeeman coherence $\sigma_W(z, t) = \sigma_{12}(z, t) + \sigma_{21}(z, t)$ during the writing phase $[\sigma_W(z, t_w)]$, as described in equation (32).

Provided that $\Gamma_{12}$ is zero or close to zero, the mapping of the input pulse is preserved for a large period of time during the so-called storage phase, whose time duration is assumed to be $t_s$. After that period of time we release the stored pulse in the so-called reading phase. This phase is started by acting on the control field $\Omega_T^R(t')$ at time $t' = 0$, where $t' = t - (t_0 + t_s)$, in such a way that $\Omega_T^R(t') = 0$. In the following analysis, the relevant magnitudes should incorporate super-index $R$ to indicate that they are considered in the reading phase, and we will replace $t'$ with $t$ to simplify the notation. In this phase, the process is described in terms of equations (25), (29) and (31). In view of the adiabatic conditions assumed, the second order derivative in equation (31) can be neglected, and after minor algebra we arrive at the following set of coupled equations for the probe field and the Zeeman coherence:

$$\frac{\partial}{\partial t} E_p^R(z, t) + c \frac{\partial}{\partial z} E_p^R(z, t) = -\frac{2 Ng^2 n^{(0)}}{\Gamma_1} E_p^R(z, t) \quad (36)$$

$$\frac{\partial}{\partial t} \sigma_R^R(z, t) = -\frac{2 \Omega_T^R n^{(0)}}{\Gamma_1} E_p^R(z, t) - \frac{2 \Omega_T^R^2}{\Gamma_1} \sigma_R^R(z, t). \quad (37)$$
In view of the linear character of equations (36) and (37), we can take advantage of using the Fourier transform in the z-space, thus obtaining a pair of coupled ordinary differential equations for the k-component of both $E_R^R(z,t)$ and $\sigma_R^R(z,t)$. The transformed magnitudes will be denoted with a hat: $\hat{\sigma}_R^R(k,t) = \mathcal{F}[\sigma_R^R(z,t)]$ and $\hat{E}_R^R(k,t) = \mathcal{F}[E_R^R(z,t)]$, respectively. The resulting equations can be casted into matrix form as

$$\frac{d}{dt} U = M U,$$

where $U(k,t) = [\hat{E}_R^R(k,t), \hat{\sigma}_R^R(k,t)]^T$, and $T$ stands for the vector transpose. $M$ is a $2 \times 2$ matrix, whose elements are

$$M_{11} = -(ikc\Gamma_1 + \frac{2k}\Omega_0^2), \quad M_{12} = -\frac{2\Omega_0^2 R_0}{\Gamma_1}, \quad M_{21} = -\frac{2\Omega_0^2 R_0}{\Gamma_1}, \quad M_{22} = -\frac{2\Omega_0^2 R_0}{\Gamma_1}.$$

The solution of equation (38) can be written as $U(k,t) = C^e e^{i\omega_1 t} e^{-\frac{2i\Omega_0^2 R_0}{\gamma_1 t}} + C^o e^{-i\omega_1 t} e^{-\frac{2i\Omega_0^2 R_0}{\gamma_1 t}}$, where $\lambda_+$ and $\lambda_-$ are the eigenvalues and eigenvectors of matrix $M$. The expression obtained for the eigenvalues to first order in $k$ are given by

$$\lambda_+ = -\frac{4\Omega_0^2 R_0}{\Gamma_1} \left(1 - \frac{M_{12} M_{21}}{2[\Omega_0^2 R_0^2 - \gamma_1^2 R_0^2]^2}\right),$$

$$\lambda_- = -\frac{4\Omega_0^2 R_0}{\Gamma_1} \left(1 + \frac{M_{12} M_{21}}{2[\Omega_0^2 R_0^2 - \gamma_1^2 R_0^2]^2}\right).$$

In view of equation (39) we conclude that the term evolving in time with $\lambda_-$ represents a decaying contribution, while that evolving with $\lambda_+$ represents a pure oscillation, which accounts for the possibility of recovering the k-component of the stored pulse.

The explicit expressions obtained for the eigenfunctions are: $\psi^e = [\phi^+, 1]^T$, where $\phi^± = -2(\Omega_0^2)^2/(2\Omega_0^2 R_0^2 + \Gamma_1)$, $\lambda_\pm$. Finally, the values for $C^e$ and $C^o$ are determined by imposing the initial condition $U(k,t = 0) = [\hat{E}_R^R(k,t = 0), \hat{\sigma}_R^R(k,t = 0)] = [0, \hat{\sigma}_R^R(k,t = 0)]$. We recall here that $\hat{\sigma}_R^R(k,t = 0) = \hat{\sigma}_R^R(k,t = 0)$, in view of the fact that we have assumed $\Gamma_2 = 0$. By taking into account all the previous considerations, the k-component of the output field is approximately given by

$$\hat{E}_R^R(k,t) = \frac{R_0^{\gamma_1}}{2\Omega_0^2 R_0}(k,t) E_R^R e^{i[kz + \lambda_+ t + \lambda_- t + \phi_1^\pm - \phi^\pm]}. $$

By transforming back to the $z$-space we obtain the pulse which is recovered during the reading phase from the medium. It is to be noted that in the X-type atom we have arrived at similar results to those obtained for $\Lambda$-type atoms in [49], where the role played by the Zeeman coherence $\sigma_R^R(z,t)$ in the present system is that associated with the sublevel coherence in the $\Lambda$-type atom.

In order to confirm these effects, we have numerically solved the full set of density matrix equations (6) and the field equation (7) without invoking any perturbative approximation once the Bloquet expansion has been made and the terms oscillating at $\delta$ are selected. We assume standard atomic initial conditions, where all the atoms are in the ground state ($\langle \sigma_{ij}(0) \rangle = 1$) and the remaining coherences vanish, i.e., $\langle \sigma_{ij}(0) \rangle = 0$ ($\forall i, j = 1, \ldots, 4$). We define the following normalized quantities: $\tau = \gamma t$, $\zeta = \gamma t$, $\tilde{E} = E / \gamma$. We consider a Gaussian type input probe field given by

$$E_p(0, \tau) = E_p(0) e^{-\frac{\tau^2}{2\tau_0^2}},$$

where $\tau_0$ and $\tau_0$ being the center and the width of input pulse, respectively. The amplitude of the input pulse $E_p(0) \ll 1$ has been chosen small enough to avoid distortions of the pulse arising from nonlinearities and/or absorption.

The time evolution of the control parameter $\Omega_\lambda$ along the different phases of the process is determined according to

$$\Omega_\lambda(t) = \Omega_{\lambda_0} \left[ \alpha_{12} - \alpha_1 \tan[f_{\Omega_\lambda}(t) - \tau_{on}] \right] + \alpha_2 \tan[f_{\Omega_\lambda}(t) - \tau_{on}]],$$

where $\Omega_{\lambda_0}$ represents the maximum variation of the control field. Besides, $\tau_{on}/\tau_{on}$ stands for the instant of time where the control parameter is switched off/on. Magnitudes $\alpha_1$ ($i = 1, 2$) are the amplitudes of the falling (preceded by a minus sign) and/or rising (preceded by a plus sign) components of the corresponding magnitude. The following condition holds: $\alpha_{12} = \alpha_1 + \alpha_2$, which guarantees that the control field parameters reach a zero value during the storage phase. The temporal variation of the control parameters given in equation (42) is similar to those previously used in the adiabatic storage of light pulses in a $\Lambda$-type atomic medium [6].

The operation mode consists in the adiabatic variation of the control parameter. Figure 6(a) presents the time variation of the classical control field during the process of writing, storage and release of a weak pulse from the medium. The corresponding spatio-temporal evolution of the field is shown in figure 6(b). It can be appreciated that the pulse vanishes during the storage phase, i.e., the time interval where the classical control field is turned off. In order to show where the electromagnetic excitation is stored we present in figure 6(c) the spatio-temporal evolution of the modulus of the average of the operator $\psi = \sigma_R^R(z,t) - \sigma_R^R(z,t)$. We observe that in the storage phase this magnitude remains nearly stationary in space during the course of time, thus demonstrating that the incoming field has been mapped into this combination of Zeeman coherences. The system mimics the behavior of a medium composed of $\Lambda$-type atoms.

Finally, it should be stressed that in the highly non-degenerate case, which corresponds to the application of a high magnetic field, the system will allow the storage operation, since we have shown that in this regime the $X$-system behaves as two quasi-independent three $\Lambda$-type systems, and exhibits nearly EIT operation, as depicted in figure 3(b).

5. Conclusions

In this work we have analyzed the linear and nonlinear optical properties of a positive charged quantum dot in the Voigt geometry for the external magnetic field. In the linear regime the system exhibits transparency at the line center in the regime of low Rabi frequencies of the control field which arise from Zeeman coherent oscillations. In the case that the
Rabi frequencies become comparable to the linewidth, the linear response resembles that of two-level systems. When the external magnetic field is turned off, we have found that the system may operate like an all-optical switching for the probe field by the external manipulation of the control field while keeping constant the rest of parameters. In addition, the system has been shown to exhibit a magneto-optical switching behavior which is slower than the all-optical switching operation. Apart from these considerations, the use of the Floquet analysis reveals that the current system exhibits nonlinear enhancement with vanishing linear absorption by properly detuning the auxiliary control field. We have also shown the feasibility of an alternative schema for storage and retrieval of weak light pulses, based on Zeeman induced coherences. Numerical solutions of the set of Maxwell–Bloch equations reveal that a weak probe field can be mapped into a spin-wave channel by adiabatically changing the auxiliary classical field, and later released from the medium on demand. The main limitation of the current proposal arises from the fact that we have neglected the effects of inhomogeneous broadening. One reason for the inhomogeneous broadening is that the energy of electrons in the valence band and the conduction band depends on the size of the QDs. Due to the nonuniformity of QDs, the signal and pump fields will experience different detunings when interacting with different dots. It should manifest as a modification in the transition frequency, as has been analyzed for example in [51] by considering a Poisson-like distribution for the sizes of the QDs. Chang-Hasnain et al [52] also have addressed this problem in the context of the slow down and storage of light pulses in a semiconductor buffer in QDs, and they showed that such system may exhibit an adequate level of storage in spite of the nonuniformity of size of the QDs. The impact of such inhomogeneities on the performance of the current system as an optical memory, and for the switching of the probe field, is out of the scope of the current study, although we recognize the necessity of further studies on this question that will be carried out and presented elsewhere. It is expected that improvements of QD linewidth and uniformity in the future can lead to the development of all-optical switches and quantum memory devices like the one described in this work.

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Appendix. Equation of motion in a degenerate X-like atomic system

In the case that the magnetic field is turned off, the system is degenerate and $\Delta_{z} = \Delta_{x} = 0$, thus the equations of motion for the Heisenberg operators can be simplified to the following set of equations

\[
\begin{align*}
\frac{\partial \hat{\sigma}_{x}}{\partial t} &= -i [\Gamma_{14} + i \Delta_{x}] \hat{\sigma}_{x} + ig E_{p} e^{-i\delta_{p}} (\hat{\sigma}_{x} - \hat{\sigma}_{y}), \\
\frac{\partial \hat{\sigma}_{y}}{\partial t} &= -i [\Gamma_{14} + i \Delta_{x}] \hat{\sigma}_{y} + ig E_{p} e^{-i\delta_{p}} (\hat{\sigma}_{x} - \hat{\sigma}_{y}), \\
\frac{\partial \hat{\sigma}_{z}}{\partial t} &= -\Gamma_{12} \hat{\sigma}_{z} - \Gamma_{13} \hat{\sigma}_{x},
\end{align*}
\]

(A.1)

where $\sigma_{T} = \sigma_{15} + \sigma_{24}$, $\sigma_{p} = \sigma_{14} + \sigma_{23}$, $\sigma_{x} = \sigma_{12} + \sigma_{31}$, $\sigma_{y} = \sigma_{13} + \sigma_{24}$, $n_e = n_{11} + n_{22}$, and $n_c = n_{33} + n_{44}$. The Langevin operators have been neglected in equation (A.1) since in the following discussion we consider the adiabatic regime.

These equations can be solved by using a Floquet expansion to first order in the field amplitude $E_{p}$ of the form

\[
\sigma_{j} = \sigma^{(0)} + g E_{p} e^{-i\delta} \sigma^{(1)} + g E_{p} e^{i\delta} \sigma^{(-1)} \quad (j = T, P, x, y).
\]

(A.2)

The wave equation for the field amplitude $E_{p}$ can be shown to be given by

\[
\frac{\partial}{\partial t} E_{p}(z, t) + c \frac{\partial}{\partial z} E_{p}(z, t) = i N g \hat{\sigma}_{p}^{(1)}.
\]

(A.3)

In view of equation (A.3) we need to find the solutions for the source term $\hat{\sigma}_{p}^{(1)}$. These equations are easily derived from equations (6) by assuming that the control field is tuned to resonance ($\Delta_{x} = 0$) and read

\[
\begin{align*}
\frac{\partial \hat{\sigma}_{p}^{(1)}}{\partial t} &= -[\Gamma_{14} - i \delta] \hat{\sigma}_{p}^{(1)} + i \Omega_{C} (\hat{\sigma}_{x}^{(1)} - \hat{\sigma}_{y}^{(1)}), \\
\frac{\partial \hat{\sigma}_{c}^{(1)}}{\partial t} &= i \delta \hat{\sigma}_{x}^{(1)} - ig E_{p} \hat{\sigma}_{c}^{(0)} + i \Omega_{C} (\hat{\sigma}_{p}^{(1)} - \hat{\sigma}_{p}^{(-1)*}), \\
\frac{\partial \hat{\sigma}_{p}^{(-1)}}{\partial t} &= -[\Gamma_{34} - i \delta] \hat{\sigma}_{p}^{(-1)} - \frac{\partial \hat{\sigma}_{x}^{(1)}}{\partial t},
\end{align*}
\]

(A.4)

The coherences $\sigma_{ij}$ are implicitly assumed to be functions of $z$ and $t$, i.e., $\sigma_{ij} = \sigma_{ij}(z, t)$.

References

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