Coherent control of light pulses stored in a Gradient Echo Memory

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We address the problem of storage and retrieval of a time-bin qubit onto an atomic ensemble of three-level atoms of the $\Lambda$-type driven by a far detuned coupling field. The atoms are subject to a longitudinal magnetic field which produces a spatially varying Zeeman splitting of the lower levels along the medium and allows for the spatial encoding of the different angular frequencies of the input pulse during the storage phase. The system operates like a quantum optical memory which is referred to as Gradient Echo Memory (GEM) in analogy to the NMR gradient echo. In the conventional approach to GEM-based devices in $\Lambda$-type media, the coupling field co-propagates with the probe field and the magnetic field is reversed after the storage phase, thus an echo signal is released from the medium in the forward direction, in contrast to $\pi$-pulse based echo memories which produce a time reversed echo signal in the backward direction. In this work we predict that a backward echo pulse can also be obtained in a GEM schema by making use of a counter-propagating coupling field during the reading phase of the process. In addition, we show that for a proper engineering of the control parameters the system can act as an optical router, i.e., part of the stored signal can be released in the forward direction and the rest in the backward one. The current proposal could have potential applications in quantum information processing.

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1. Introduction

The storage of quantum states of light in different atomic ensembles has become a field of intensive research pursuing the development of quantum memory (QM) devices for on-demand recall of the stored states with fidelity beyond the classical limit. The use of atomic ensembles to map classical as well as non-classical light pulses has been proposed [1–3] and experimentally verified [4–6]. To this end different atomic schemas [7–12] have been considered (see also the review in Ref. [13]). Some proposals based upon electromagnetically induced transparency (EIT) have been considered to realize such a memory device in ensembles of $\Lambda$-type atoms and other multilevel configurations. The underlying physics of an EIT-based QM can be described in the language of combined field-matter operators by introducing the so-called polariton operator which is an admixture of matter and field operators. The system works in the low excitation limit and adiabatic following, and allows one to map the incoming light pulse onto a long-lived spin coherence for on-demand recall. The quantum properties of the stored light have been shown to be well preserved [14–16].

Other protocol relies on the so-called photon echo effect [17] which was initially addressed in a series of papers [18–22]. This storage technique has received a renewed attention after the proposal by Moiseev and Kröll [23] and it makes use of a counter-propagating $\pi$ pulse (reading pulse) which enables to release the light field stored in the medium with an efficiency higher than that obtained in the conventional approach. The $\pi$ pulse induces a re-phasing of the atomic dipoles and cancels out the dephasing produced during the storage phase. The performances of this proposal have been addressed in detail in gasses [24] as well as in solid state media [25,26]. Other photon-echo based proposal for a QM makes use of an inhomogeneously broadened optical transition which is shaped into an atomic frequency comb (AFC) by means of selective optical pumping to an auxiliary level [27,28]. The control of the broadening of the absorption line by means of external magnetic or electric fields has been proposed to produce the rephasing of the atomic ensemble [29]. This alternative has been experimentally verified by Alexander et al. [30] in a rare earth doped medium subject to a longitudinal varying electric field. Such a system will operate as an electro-optic memory or Gradient Echo Memory (GEM) and should exhibit a theoretical maximum efficiency close to 100% with high fidelity and a large time-bandwidth product [31–34]. A recall efficiency as high as 26% was experimentally achieved [31]. The GEM was also shown to be feasible by considering a far detuned Raman resonance in the $^{87}$Rb line broadened by a longitudinal varying magnetic field which was reversed in time [35]. The system was described by three-level model of the $\Lambda$-type and recall efficiencies up to 30% were experimentally achieved. The selective release of the stored light pulses in such medium has been verified [36] by allowing for multiple reversion of the magnetic field accompanied by turning on and off the coupling field. The engineering of the rephasing process has been shown to provide an additional degree of freedom which allows for the
frequency shifting, spectral compression and fine dispersion control over the pulses released from the memory [37]. Recent experiments conducted in this system report recall efficiencies as large as 87% [38]. The analysis of noise characteristics of GEM based devices has been shown to allow for storage and recall of weak coherent states more faithfully than a classical memory [39]. The use of the gradient echo process in an ensemble of four-level atoms of the tripod type has been shown to allow for all-optical quantum multiplexing of two trains of pulses [40]. A similar approach for quantum state storage makes use of controlled Stark shifts in rare earth doped crystalline and amorphous waveguides [41–44] which has lead to the development of a solid state quantum memory operating in the telecommunication window [45]. We refer the reader to the recent reviews [46,47] for a detailed description of the state of the art in the development of QMs.

The aim of this paper is to extend the proposals by Héret et al. [35] and Hosseini et al. [36,38]. In all of these works a GEM device was shown to operate in a medium of $A-$type atoms by making use of a far-off-resonance Raman coupling to the ground states, thus a quantum field could be mapped onto the lower levels’ coherence with the use of an auxiliary classical control field. This coherence exhibits a weak decay rate and allows lengthening the duration of the storage phase. The up-to-date schema has made use of a classical field which co-propagates with the quantum field along the whole process: mapping of the incoming pulse, storage, and release of the echo pulse from the medium. In this operation mode the rephasing of the atomic dipoles produces a forward echo pulse. In the present work, we consider the use of a co-propagating auxiliary classical pulse during the writing phase, while the propagation direction of this classical field can be selectively changed during the reading phase of the process. This change accompanied by the simultaneous reversion of the magnetic field will result in the rephasing of the atomic dipoles which in turn will produce a backward echo pulse which is a time-reversed copy of the stored pulses. Thus the GEM device shows a mode of operation which resembles to that of quantum memories while a far detuned classical field drives transition $|1\rangle \leftrightarrow |2\rangle$ and the optical coherence ($\sigma_{13}$) is not required. In addition, we show that a proper engineering of the coupling fields allows the system to operate like an optical router, i.e., part of the stored signal can be released in the forward direction and the rest in the backward one, although this requires an $a$ priori knowledge of the temporal shape of the input pulse. We select a gaseous medium like $^{87}$Rb since its use will allow us to compare our results with those previously reported on this medium, although we are aware about the hampering factor for short storage time due to the loss of atoms from the interaction region. It is worth mentioning that the obtention of photon echoes which relies in the control of the inhomogeneous broadening in solid state system has been probed in Eu$^{3+}$:Y$_2$SiO$_5$ by making use of an external electric field gradient [30], thus the time reversal of inhomogeneous broadening of the optical transition does not pose major problems in a Gradient Echo Memory. Note also that a similar procedure has been used to obtain photon echoes in an Erbium doped silicate fiber to obtain a GEM-based device [45]. The paper is organized as follows: Section 2 lays the groundwork model needed to illustrate our approach, which is then used in Section 3 to present numerical simulations which confirm the theoretical predictions. Section 4 summarizes the main conclusions.

2. Theoretical model and equations

We consider an ensemble of $A$-type three level atoms comprising an excited state $|3\rangle$ and two lower states $|1\rangle$ and $|2\rangle$. A probe field of angular frequency $\omega_p$ and wave vector $k_p$ drives transition $|1\rangle \leftrightarrow |3\rangle$ while a far detuned classical field of angular frequency $\omega_c$ and wave vector $k_c$ drives transition $|2\rangle \leftrightarrow |3\rangle$ (see Fig. 1). This schema has been experimentally addressed through the $D_1$ line in $^{87}$Rb in Refs. [35–38]. A longitudinal magnetic field $\mathbf{B}$ is applied along the medium to create a linearly varying frequency lower levels’ splitting by means of wound magnetic coils surrounding a gas cell [35]. The different angular frequencies of the incoming probe signal are resonant with the atoms at different locations of the sample, thus producing a spatial encoding of the spectral components of the incident optical field. The storage of a weak pulse in the medium can be analyzed by considering the propagation along a single spatial dimension, let’s say the $z$ axis, while neglecting spatial transverse effects. The quantum/classical field propagates in the positive $z$ direction of a medium of length $L$ with amplitude $\hat{E}_p(z, t)/\epsilon_0(z, t)$. The dynamics of atomic populations and coherences for this level configuration can be derived through standard procedures, which in the rotating-wave and electric dipole approximations, allows one to derive a set of Heisenberg-Liouvillian equations for the atomic operators [48]. We also assume the weak-field limit which assumes that population remains essentially in the ground state, thus $\sigma_{11} \approx 1$, while $\sigma_{22} \approx \sigma_{33} \approx 0$, $\sigma_{12}^\dagger / \sigma_{12}^\ast$ being the identity/null operator. In this situation the relevant equations describing the time evolution of the lower levels’ coherence ($\sigma_{12}$) and the optical coherence ($\sigma_{13}$) reduce to

$$\frac{\partial \sigma_{12}}{\partial t} = - (\gamma + i\delta(z, t))\sigma_{12} + i\Omega_c^*\sigma_{13} + F_{12}(z, t),$$

$$\frac{\partial \sigma_{13}}{\partial t} = - (\gamma + \gamma_0 / 2 + i\Delta)\sigma_{13} + ig\hat{E}_p + i\Omega_c\sigma_{12} + F_{13}(z, t),$$

\(\gamma\) being the upper level decay rate and $\gamma_0$ represents the ground state decoherence. In addition, $g$, $\Omega_c$, and $\Delta$ stand for the matter–coupling constant for the quantum field, and the Rabi frequency and the detuning of the coupling field, respectively. The lower levels’ splitting exhibits a linear dependence, i.e., $\delta(z, t) = \eta(t)z$ and the sign of $\eta$ can be reversed by acting on the current of the coils [35]. The Langevin operators $F_{12}$ and $F_{13}$ account from noise coming from spontaneous emission and ground state decoherence. In view of the fact that these
processes do not add excess noise, the Langevin terms can be neglected in the subsequent analysis.

The equation of coherences [Eq. (1)] is complemented with the Maxwell wave equation for the quantum field which in the slowly varying amplitude approximation reads
\[
\frac{\partial \epsilon_p(z,t)}{\partial t} + c \frac{\partial \epsilon_p(z,t)}{\partial z} = i N g \sigma_{13},
\]
(2)
with \(N\) being the atomic density.

The coupling field is allowed to be either in the forward (FW) or backward (BW) propagation direction. In this case the Rabi frequency of this field be expressed in the general form as
\[
\Omega_c(z,t) = \Omega_+ \sigma(t) e^{i \delta z} + \Omega_- \sigma(t) e^{-i \delta z},
\]
(3)
with \(\Omega_+\sigma(t)\Omega_-\sigma(t)\) being the slowly varying Rabi frequency of the FW/BW component of the classical field. In the presence of such coupling field it is convenient to decompose the probe field into two slowly varying components \(\epsilon_{p+}\) and \(\epsilon_{p-}\) of the form
\[
\epsilon_p(z,t) = \epsilon_{p+}(t) e^{i \delta z} + \epsilon_{p-}(t) e^{-i \delta z}.
\]
(4)

In view of these considerations, we make use of an expansion of the coherences \(\sigma_{12}\) and \(\sigma_{13}\) in their spatial Fourier components according to
\[
\sigma_{12}(z,t) = \sum_n \sigma_{12}^{(0)}(t) e^{i \delta n},
\]
\[
\sigma_{13}(z,t) = \sum_n \sigma_{13}^{(0)}(2n + 1) \Omega_\pm \sigma_{13}^{(0)}(t) e^{i \delta (2n + 1)},
\]
(5)
\(n\) being an integer. The previous ansatz is substituted back into Eq. (1) and we derive an infinite set of coupled equations for the corresponding Fourier components. We truncate the solution at \(n = 0\) and arrive at the following equation for the coherences
\[
\frac{\partial \sigma_{12}^{(0)}}{\partial t} = - (\gamma_0 + i \eta) \sigma_{12}^{(0)} + i \Omega_+ \sigma_{13}^{(1)} + i \Omega_- \sigma_{13}^{(-1)},
\]
\[
\frac{\partial \sigma_{12}^{(1)}}{\partial t} = - (\gamma + \gamma_0 / 2 + i \Delta) \sigma_{12}^{(1)} + i \sigma_{13}^{(0)} + i \Omega + \Omega_- \sigma_{13}^{(-1)},
\]
\[
\frac{\partial \sigma_{13}^{(-1)}}{\partial t} = - (\gamma + \gamma_0 / 2 + i \Delta) \sigma_{13}^{(-1)} + i \sigma_{13}^{(0)} + i \Omega_+ \sigma_{12}^{(0)},
\]
(6)
whereas the equations for the components of the probe field reduce to
\[
\frac{\partial \epsilon_{p+}(z,t)}{\partial t} + c \frac{\partial \epsilon_{p+}(z,t)}{\partial z} = -i (\alpha_e - \alpha_p) \epsilon_{p+} + i N g \sigma_{13}^{(1)},
\]
\[
\frac{\partial \epsilon_{p-}(z,t)}{\partial t} - c \frac{\partial \epsilon_{p-}(z,t)}{\partial z} = -i (\alpha_e - \alpha_p) \epsilon_{p-} + i N g \sigma_{13}^{(-1)}.
\]
(7)

The previous analysis has been applied in the context of studying stationary light pulses in atomic media [49]. Note that \(\sigma_{13}^{(1)} / \sigma_{13}^{(-1)}\) in Eq. (7) acts as a source for the probe field component \(\epsilon_{p+} / \epsilon_{p-}\). In what follows we assume that the angular frequencies of the probe and coupling fields are nearly equal, thus the angular frequency mismatch of the first term in the right-hand side of Eq. (7) cancels out.

The set of Eqs. (6) and (7) can be further simplified: to this end we adiabatically eliminate the fast excited-state fluctuations by considering that the inverse of the fastest time scale of the process is negligible compared to the decay rate \(\gamma\). In addition we assume a large detuning \(\Delta\) compared to the spontaneous emission rate \((\Delta \gg \gamma)\). Thus, Eq. (6) reduces to
\[
\frac{\partial \sigma_{12}^{(0)}}{\partial t} = - \left( \gamma_0 + i \eta z + i \frac{\Omega_+ + \Omega_-^2 + \Omega_-^2}{\Delta} \right) \sigma_{12}^{(0)} + \frac{i \Omega + \Omega_-}{\Delta} \epsilon_{p+} + \frac{i \Omega_+}{\Delta} \epsilon_{p-}.
\]
(8)

In the case of Eq. (7) we make use of the moving frames for the forward and backward probe fields which produces the following result
\[
\frac{\partial \epsilon_{p+}(z)}{\partial z} = - i N g \frac{\Omega_+ \sigma_{13}^{(0)}}{\Delta},
\]
\[
\frac{\partial \epsilon_{p-}(z)}{\partial z} = - i N g \frac{\Omega_- \sigma_{13}^{(0)}}{\Delta},
\]
(9)
where \(z = z - t / c\), and \(\tilde{z} = z + t / c\). It is to be noted that the reference frames have been normalized to the effective refractive index \(i g N / \Delta\).

The process of storage and retrieval of the quantum field can be divided into three phases: during the first part (the writing phase: WP) a gradient frequency is established within the sample and the control field is turned on and co-propagates with the quantum field. The gradient frequency and the control field are reset to zero at second stage (the storage phase: SP). Finally, the gradient frequency is reversed while the control field is again turned on, during the so-called reading phase (RP). The three phases of the process are indicated by the dashed vertical lines in Fig. 2(a). The turning off of the control field during the storage phase results in a greater recall efficiency than in the case when the control field is maintained along the whole process [38]. In the conventional approach, \(\Omega_\pm(t) = 0\), thus only the optical coherence \(\sigma_{13}^{(1)}\) is excited along the whole process and, consequently, no backward echo signal is obtained \(\epsilon_{p-}(z', t) = 0\) [see also Eqs. (8) and (10)], whereas the reversion of the detuning slope \((\nu \rightarrow -\nu)\) once a partial absorption of the input pulse has been achieved, results in the production of an echo in the forward direction which is a time-reversed copy of the input pulse [35].

However, we can devise another reading procedure by using a counter-propagating coupling field during the reading phase: a close inspection of the evolution equation for \(\sigma_{13}^{(1)}\) reveals that the use of a counter-propagating coupling fields allows to excite this coherence which is also coupled to the long-lived coherence \(\sigma_{13}^{(2)}\), thus resulting in the generation of a backward propagating echo pulse: \(\epsilon_{p-}(z', t) \neq 0\).

3. Light storage and retrieval

In order to confirm the effects mentioned in the previous discussion, we numerically solve the averaged Heisenberg Eq. (8) and the field Eqs. (9) and (10). We assume standard atomic initial conditions \((\sigma_{12}^{(0)} = 0)\). We are concerned with encoding of quantum information into qubits, that is, quantum states that are described by
\[
|\psi > = \alpha |0 > + \beta e^{i \phi} |1 >,
\]
(11)
where the coefficients \(\alpha, \beta\) and \(\phi\) are real and satisfy \(\alpha^2 + |\beta|^2 = 1\), whereas the kets \(|0\rangle\) and \(|1\rangle\) form an orthonormal basis in a two-dimensional Hilbert space. In particular, we are interested in the so-called time-bin qubits, where the states \(|0\rangle\) and \(|1\rangle\) describe photon wave packets localized at particular positions.

Time-bin qubits have
the beam has a diameter of 0.3/2 cm. It has been shown to be suited for quantum computation \[50\]. The problem of encoding quantum information into photonic qubits has been addressed recently in Ref. \[51\], showing that the qubits remain

\begin{align*}
\eta(t) &= \eta_0 \left(-0.5 \tanh[0.6(t-t_0)] - 0.5 \tanh[0.6(t-t_1)]\right), \\
\Omega_{\pm}(t) &= \Omega_0 \left(1.0 - 0.5 \tanh[0.6(t-t_0)] + 0.5 \tanh[0.6(t-t_1)]\right),
\end{align*}

thus the storage phase lasts \(t_1 - t_0\).

Fig. 2(a) presents the time evolution of the control parameters used in the simulation: solid/dashed line corresponds to the time evolution of the frequency gradient/forward coupling field. Fig. 2(b) displays the time evolution of the probe field during the process of writing, storage and release phases of the process. It can be appreciated that the incoming pulse experiences a partial absorption during the propagation along the medium and they are later released once the sign of the longitudinal splitting is changed at \(t_1\), which allows for the rephasing of the atomic dipoles. It is to be noted that the output echo pulse is a time-reversed copy of the input pulse as shown in Fig. 2(c).

Now we consider the case where a forward coupling field is used during the writing phase while a backward coupling field is used to release the stored excitation from the medium. In this situation the time variation of the frequency gradient follows Eq. (13) whereas the coupling fields obey the following relations

\begin{align*}
\Omega_{\pm}(t) &= \Omega_0 \left(0.5 - 0.5 \tanh[0.6(t-t_0)]\right), \\
\Omega_{\mp}(t) &= \Omega_0 \left(0.5 + 0.5 \tanh[0.6(t-t_1)]\right).
\end{align*}

This results in the transfer of the incoming pulse onto the long-lived coherence \(|\Psi_{12}\rangle\) through the use of the forward coupling field and the recall by the backward coupling field. Fig. 3(a) displays the time evolution of the control parameters used in the simulation: dashed/dashed–dotted line corresponds to the time evolution of the forward/backward coupling field, whereas solid line is used for the frequency gradient.

The result for the forward/backward probe field is depicted in Fig. 3(b)/(c) which confirms the prediction that the use of a backward propagating coupling field results in the release of a backward propagating echo pulse. Note that a single reversion of the magnetic field in Fig. 2 results in a first-input last-output mode of operation. This situation also applies in the case of the backward echo pulse for a single reversion of the magnetic field as depicted in Fig. 3(c). The efficiency of the retrieval can be computed by dividing the energy of the output pulse with regard to that of the input pulse and results in 31.4% for either the forward and the backward retrieval modes of
operation. This efficiency can be increased by changing for example
the power of the coupling field: the use of a coupling field with a
power three times that used to produce Figs. 2 and 3 results in an
efficiency up to 60.4%.

Finally, we address another retrieval mode which becomes possible by a proper engineering of the coupling fields. In this

mode, the recall of one of the input pulses is made in one direction
whereas the other pulse is retrieved in the counter-direction. To this
end, we depict in Fig. 4(a) the time evolution of the control
parameters. Note that the forward and backward coupling fields do
not coexist simultaneously within the sample: such situation will
result in the establishment of forward and backward probe fields

Fig. 3. (a) Time evolution of the frequency gradient η(t) (solid line) and the coupling field Ω₊₀(t)/Ω₋₀(t) (dashed/dashed dotted line). (b)/(c) Dynamical evolution of the modulus of the forward/backward probe field \( \varepsilon_+ (z,t)/\varepsilon_- (z,t) \). The rest of parameters as in Fig. 2.

Fig. 4. (a) Time evolution of the frequency gradient η(t) (solid line) and the coupling field Ω₊₀(t)/Ω₋₀(t) (dashed/dashed dotted line). (b)/(c) Dynamical evolution of the modulus of the forward/backward probe field \( \varepsilon_+ (z,t)/\varepsilon_- (z,t) \). The rest of parameters as in Fig. 2.
which will bounce forward and backward within the dynamical bandgap arising from the interference of both coupling fields. Such situation is beyond the scope of the current work. With this choice for the control parameters, the first input pulse is released in the forward direction while the second one is recalled in the backward direction, as it is illustrated in Fig. 4(b)–(c). Thus, the system acts as an optical-router.

4. Conclusions

We have demonstrated the feasibility of storing quantum information into time-bin qubits in a medium composed of $A$-type atoms subject to a longitudinal magnetic field which can be reversed in time either by using a forward or a backward coupling field which is used to transfer the electromagnetic excitation into a spin coherence and to recall the qubit later on demand. We have provided a physical picture for the behavior of the system and have substantiated the analytical predictions with numerical calculations that show excellent agreement with the ideal transfer process of time-bin qubits to a spin coherence. The recall efficiency for the forward/backward echo pulses is shown to be dependent on the power of the coupling field and, under proper selection of the operational parameters, allows the achievement of values which go beyond the non-cloning regime. In addition, we show that for a proper engineering of the control parameters during the reading phase, the system operates like an optical router. The current proposal adds a new mode of operation to the known implementation of a GEM-device in an ensemble of optical router. The current proposal adds a new mode of operation to the known implementation of a GEM-device in an ensemble of optical router.

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