Control of the inversionless gain and refractive index in a V-type atom via squeezed vacuum and quantum interference

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In this paper we analyze the steady-state populations and gain lineshape of a V-type three-level atom with a closely spaced excited doublet. The atom is driven by a strong coherent field, a weak probe, and a single broadband squeezed vacuum. We focus our attention in the interplay between the quantum interference and the squeezed field on the probe gain. It is shown that the relative phases between the two coherent fields and the squeezed field play an important role in the optical properties of the atom. Specifically, we find that the probe can experience gain without population inversion for proper values of the parameters characterizing the squeezed field and in the absence of incoherent pumping. The system can be tailored to exhibit multiple dispersion regimes accompanied by negligible gain or absorption over a large bandwidth, a desirable feature for obtaining propagation of pulses with negligible distortion.

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I. INTRODUCTION

In recent years a lot of interest has been shown in studying gain and lasing action without population inversion [1–3]. This effect arises from an interference between different channels, which reduces the absorption of lower-level atoms in a certain frequency range without affecting significantly the stimulated emission from upper-level atoms. Several theoretical proposals of atomic systems have appeared where coupling to one or more coherent fields allows one to manipulate the features of the fluorescence spectrum [4]. The physical essentials of the atomic coherence effects can be attributed to laser-induced dressed states as well as quantum interference resulting from different transitions pathways. Many phenomena, such as electromagnetically induced transparency [5,6], lasing without inversion [7–9], refractive-index enhancement without absorption [10–12], and giant nonlinearity [13–15], have been predicted and experimentally demonstrated and have modified the way we look at photon absorption and emission processes. However, another way of generating coherence connects with relaxation processes such as spontaneous emission, i.e., vacuum induced coherence (VIC). In an early study, Agarwal [16] showed that an initially excited degenerate V-type three-level atom may not decay to its ground level due to a cancellation of spontaneous emission by quantum interference between the atomic transitions. If the two upper levels are very close and damped by the usual vacuum interactions, spontaneous emission cancellation can take place [17–20], which offers the possibility to trap population in the excited levels when some particular conditions hold [17–22]. Quantum interference between the two transition channels connecting the ground level is responsible for many novel effects, such as narrow resonances and probe transparency [23,24], dark spectral lines [20,25], phase-dependent line shapes [26,27], gain features on dark transitions [28], vacuum-induced optical bistability [29,30], etc. The strength of the quantum interference is measured by the parameter $p = (\mu_a | \mu_b)/(|\tilde{\mu}_a| |\tilde{\mu}_b|)$, where $\mu_a$ and $\mu_b$ are the dipole moments for the two close lying states decaying to a common final state. This means that quantum interference may occur only if the dipole moments of the transitions involved are nonorthogonal. This represents a practical problem since it is very unlikely to find atoms with nonorthogonal dipole moments and quantum states close in energy [31]. However, several new methods to bypass this restriction have been proposed [32] and reviewed in an excellent work by Ficek and Swain [33]. Furthermore, Agarwal and Patnaik [34] and Gao-xiang et al. [35] have recently shown that the anisotropy of the vacuum of the electromagnetic field could lead to quantum interference among the decay channels of closely lying states even if the dipole matrix elements were orthogonal. Although the quantum interference is maximum when the transition moments are parallel, recent researches reveal novel quantum interference phenomena whose appearance does not require parallel dipole moments. For example, Paspalakis and Knight [26,36] and Dong and Sing [37] have demonstrated that in the case $p < 1$ the subnatural linewidths and the cancellation of fluorescence can depend on the relative phase between the driving fields. Gong et al. have shown that, for large values of the parameter $p$, gain can be created in two symmetric regions centered around zero probe detuning [38]. This gain for $p = 0.96$ is about eight times larger than the gain obtained in the case with no quantum interference ($p = 0$). But this gain is associated with population inversion in the transition so they obtain lasing with population inversion.

It is well known that the control of emission properties of atoms may be achieved by changing the environment so that the atoms interact with a modified set of vacuum modes. One way is to place the coherently driven atom inside a cavity, for which spectral features can be changed dramatically, for example, dynamical suppression and enhancement of the Mollow triplet have been predicted [39]. Another method is to bath the atom in a squeezed vacuum [40,41]. The interaction of atomic systems with squeezed light has been a subject of intense activity since Gardner showed that the two dipole quadratures of a two-level atom interacting with a squeezed vacuum decay at different rates [40]. This phenomenon leads to the prediction of a number of different effects, and a variety of classic and standard problems have been reexamined with squeezed light. Carmichael et al. [42] showed that the
resonance fluorescence spectrum of a resonantly driven two-level atom exhibits a Mollow-like triplet for strong driving fields, as it happens in the standard vacuum [43]. However, the linewidth of the central peak strongly depends on the relative phase of the driving field and the squeezed vacuum. In the context of gain without population inversion, Ficek et al. [44] have shown that a two-level atom damped by a broadband squeezed vacuum can exhibit a strong emission peak (gain) at the central frequency of the atomic levels, which is not attributed to population inversion, and results from the so-called coherent population oscillations [45]. The effects of a broadband squeezed vacuum on a three-level atom in A, V and ladder configurations have also been analyzed [30,46–49]. The resonance fluorescence spectra of a three-level atom which interacts with two coherent lasers and two independent squeezed vacua have been studied [50–52]. Ferguson et al. [51] have examined the fluorescence spectrum for a strongly driven three-level system in which one of the two one-photon transitions is coupled to a finite-bandwidth squeezed vacuum field. Bosticky et al. [52] have considered the behavior of a three-level atom in a ladder configuration with the lower transition coherently driven and damped by a narrow-bandwidth squeezed vacuum field. Great attention has been paid to the effect of a squeezed vacuum on the two-photon excitation rate for a three-level atom in the ladder configuration, where a linear dependence of the excitation rate on the excitation intensity in the weak-field limit is predicted, instead of a quadratic one for classical excitation field [53,54].

In this work we study the interaction of closely spaced doublet V-type atoms with a squeezed vacuum. We assume that the bandwidth of the squeezed vacuum is much larger than the natural linewidth of the transitions, that is, a single broadband squeezed vacuum is coupled to both transitions. Therefore, we consider that squeezed vacuum-induced interference among decay channels is significant. In particular, we are interested in the interplay between quantum interference and the squeezed vacuum on the optical properties of the atom. We also consider the situation where the atom is driven by two monochromatic coherent fields. The arrangement is such that each field couples only one transition in a pump-probe configuration. This is the main difference with previous studies [26,27]. We show that the system becomes sensitive to the amplitude and the phase of the squeezed field. The steady-state population and the gain of the weak probe field is investigated. A remarkable effect induced by the squeezed field is the presence of gain without population inversion in the system. We also find that the squeezed field induces a notable influence on the refraction index. For suitable values of the squeezed field parameters we found a nearly zero plateau in both the absorption and dispersion curves.

The paper is organized as follows. Section II establishes the model, i.e., the Hamiltonian of the system and the evolution equation of the atomic operators assuming the rotating wave approximation. Section III is devoted to discussing the effect of the squeezed vacuum on the steady-state population of the atomic levels and the gain of the probe field. In Sec. IV the absorption and gain features are explained in terms of the atomic population distribution in the dressed states. The influence of the squeezed vacuum in the refraction index is analyzed in Sec. V. Finally, Sec. VI provides the conclusions.

II. ATOMIC MODEL AND DENSITY-MATRIX EQUATIONS

We consider a V-type three-level atom with two near-degenerate excited levels |2⟩ and |3⟩, and a ground level |1⟩ as shown in Fig. 1(a). We study the situation where the pump and probe fields act on two different arms of the V system: the strong pump field \( \vec{E}_2 \) only drives the \( |2⟩ \rightarrow |1⟩ \) transition to prepare population distributions in the excited levels, and similarly the probe field \( \vec{E}_3 \) only drives the \( |3⟩ \rightarrow |1⟩ \) transition, so that the transition dipole moments \( \mu_{31} \) and \( \mu_{21} \) are nonorthogonal. Furthermore, the pump field will be considered to be on resonance with the \( |2⟩ \rightarrow |1⟩ \) transition, and both the pump and the driving field are in phase with each other. The energy-level scheme is shown in Fig. 1 together with the arrangement of the field polarizations [55]. We just consider both fields to be linearly polarized with the restrictions \( \mu_{31} \cdot \vec{E}_2 = 0 \) and \( \mu_{21} \cdot \vec{E}_3 = 0 \) and with the same angular frequency. Therefore, the total field is given by

\[
\vec{E} = \vec{E}_2 + \vec{E}_3 = \frac{1}{2} \vec{E}_{20} e^{-i(\omega_L t + \phi)} + \text{c.c.} + \frac{1}{2} \vec{E}_{30} e^{-i(\omega_L t + \phi)} + \text{c.c.},
\]

(2.1)
\( \hat{E}_{j0} (j = 2,3) \) being the amplitudes of the slowly varying field envelopes, and \( \phi \) and \( \omega_L \) the phase and angular frequency of the fields, respectively. Spontaneous and stimulated emissions between these states are governed by the interaction of the atom with a reservoir in a multimode squeezed state. In order to take into account the induced-coherence effects by spontaneous emission, the two upper levels \([3,2]\) are coupled by the same vacuum modes to the ground level \([1]\). The resonant frequencies between the upper levels \([3,2]\) and the ground level \([1]\) are \( \omega_{31}, \omega_{21} \), respectively. Note that \( \omega_{31} - \omega_{21} = \omega_{32}, \omega_{32} \) being the frequency separation of the excited levels. The Hamiltonian of the system in the rotating wave approximation is given by \([16,56,57]\)

\[
H = \hbar \omega_m |m\rangle \langle m| + \hbar \sum_{k,k'} \omega_{k\lambda} a_{k\lambda}^\dagger a_{k\lambda} + \hbar \sum_{m = 2}^{3} \sum_{k,k'} g_{mk} |m\rangle \langle 1| a_{k\lambda} + \text{H.c.} \\
- \hbar \sum_{m = 2}^{3} \Omega_m e^{-i (\omega_L t + \phi)} |m\rangle \langle 1| - \text{H.c.},
\]

(2.2)

\( \hbar \omega_m \) being the energies of the atomic levels, \( a_{k\lambda} \) \((a_{k\lambda}^\dagger)\) is the annihilation (creation) operator of the \( k \)th mode of the vacuum field with polarization \( e_{k\lambda} \) \( (\lambda = 1,2) \) and angular frequency \( \omega_{k\lambda} \). The parameter \( g_{mk} \) is the coupling constant of the atomic transition \(|m\rangle \rightarrow |1\rangle \) with the electromagnetic mode

\[
g_{mk} = - \sqrt{\frac{\omega_{k\lambda}}{2\hbar e_0}} V(a_{1\lambda}^\dagger - e_{k\lambda}^*),
\]

(2.3)

where \( \tilde{\mu}_{1m} \) is the dipolar moment of the transition and \( \Omega_m = \tilde{\mu}_{1m} \tilde{E}_{m0}/(2 \hbar) \) is the Rabi frequency of the transition \(|m\rangle \rightarrow |1\rangle \). By taking into account the polarization arrangement shown in Fig. 1(b), we arrive at the following relations between the normalized fields and the quantum interference parameter \( p = \mu_{13}^* \mu_{12}/(\mu_{13} |\mu_{12}|) = \cos(\theta) \):

\[
\Omega_2 = \frac{\tilde{\mu}_{12} |\tilde{E}_{20}|}{2\hbar} (1 - p^2) = \Omega_{20} (1 - p^2).
\]

We now assume that the quantized radiation field is in a broadband squeezed vacuum state with carrier frequency \( \omega_c \), which is tuned close to the frequency of the atomic transitions \([3\rightarrow 1]\) and \([2\rightarrow 1]\), that is, \( 2\omega_c = \omega_{31} + \omega_{21} \). The bandwidth of the squeezing is assumed to be broad enough so that the squeezed vacuum appears as \( \delta \)-correlated squeezed white noise to the atom. Besides, the bandwidth of the squeezed field is larger than the upper level spacing. The correlation function for the field operators \( a(\omega_{k\lambda}) \) and \( a^\dagger(\omega_{k\lambda}) \) can be written as \([40,46]\)

\[
\langle a(\omega_{k\lambda}) a^\dagger(\omega_{k'\lambda'}) \rangle = [N(\omega_{k\lambda}) + 1] \delta(\omega_{k\lambda} - \omega_{k'\lambda'}),
\]

\[
\langle a^\dagger(\omega_{k\lambda}) a(\omega_{k'\lambda'}) \rangle = N(\omega_{k\lambda}) \delta(\omega_{k\lambda} - \omega_{k'\lambda'}),
\]

\[
\langle a(\omega_{k\lambda}) a(\omega_{k'\lambda'}) \rangle = M(\omega_{k\lambda}) \delta(2\omega_c - \omega_{k\lambda} - \omega_{k'\lambda'}),
\]

(2.5)

where \( N(\omega_{k\lambda}) \) and \( M(\omega_{k\lambda}) \) are slowly varying functions of the frequency and characterize the squeezing. The following inequality holds:

\[
[M(\omega_{k\lambda})]^2 \leq N(\omega_{k\lambda}) N(2\omega_c - \omega_{k\lambda}) + \text{min}[N(\omega_{k\lambda}), N(2\omega_c - \omega_{k\lambda})].
\]

(2.6)

Note that \( M \) is a complex magnitude so that \( M(\omega_{k\lambda}) = |M(\omega_{k\lambda})| e^{i\phi_c} \), where \( \phi_c \) is the phase of the squeezed vacuum. For \( M(\omega_{k\lambda}) = 0 \), Eq. (2.5) describes a thermal field at a temperature \( T_c \), where \( N(\omega_{k\lambda}) \) is the mean occupation number of the mode \( k\lambda \) with frequency \( \omega_{k\lambda} \).

The system is studied using the density-matrix formalism. By following the traditional approach of Weisskopf and Wigner \([16,56,57]\), we obtain the master equation for the reduced density matrix of the atomic system, \( \rho_j^f \), in the Born and Markov approximation. In the interaction picture the master equation reads

\[
\frac{\partial \rho_j^f}{\partial t} = -i \frac{\hbar}{\mu} [H_{ex}^f, \rho_j^f] - \frac{1}{2} \sum_{i,j=1}^{3} [N(\omega_{i1}) + 1] \gamma_{ij} [(S_j^+ S_j^- \rho_i - S_j^- S_j^+ \rho_i) e^{i\omega_{ij} t} + (\rho_i S_j^+ S_j^- - S_j^- \rho_i S_j^+) e^{-i\omega_{ij} t}]
\]

\[
- \frac{1}{2} \sum_{i,j=1}^{3} N(\omega_{i1}) \gamma_{ij} [(S_j^+ S_j^- \rho_i - S_j^- S_j^+ \rho_i) e^{i\omega_{ij} t} + (\rho_i S_j^+ S_j^- - S_j^- \rho_i S_j^+) e^{-i\omega_{ij} t}] - \frac{1}{2} \sum_{i,j=1}^{3} M(\omega_{i1}) \eta_{ij} [(S_j^+ \rho_i S_j^+) e^{i\omega_{ij} t} - S_j^+ \rho_i S_j^+ e^{-i\omega_{ij} t}]
\]

\[
- S_j^+ \rho_i^f S_j^- + (S_j^+ \rho_i^f S_j^- - \rho_i^f S_j^+ S_j^-) e^{-i(2\omega_c - \omega_{i1} - \omega_{j1}) t} - \frac{1}{2} \sum_{i,j=1}^{3} M(\omega_{i1}) \eta_{ij}^* [(S_j^+ \rho_i S_j^- - S_j^+ S_j^- \rho_i^f) + (S_j^+ \rho_i S_j^- - S_j^+ S_j^- \rho_i^f)]
\]
In the above equation we have introduced the usual shorter notation for the atomic operators [46], i.e.,

\[ S_1^i = (S_1^i)^* = |2\rangle\langle 1|, \]
\[ S_3^i = (S_3^i)^* = |3\rangle\langle 1|. \]  
(2.8)

The coefficients \( \gamma_{ij} \) and \( \eta_{ij} \) are defined as [46]

\[ \gamma_{ij} = \pi g_i(\omega_{ij}) g_j(\omega_{ij}), \]
\[ \eta_{ij} = \pi g_i(\omega_{ij}) g_j(2\omega - \omega_{ij}). \]  
(2.9)

The coefficients \( \gamma_{ii} \) in Eq. (2.7) are the decay rates for the \(|3\rangle \rightarrow |1\rangle\) and \(|2\rangle \rightarrow |1\rangle\) transitions. The additional damping terms \( \gamma_{ij} \) (\( i \neq j \)) are particularly important when \( \omega_{ij} = \gamma_2, \gamma_3 \), and they arise due to the coupling of the two transitions \(|3\rangle \rightarrow |1\rangle\) and \(|2\rangle \rightarrow |1\rangle\) with the same vacuum mode. They are responsible for the quantum interference between the two decay channels [51,58,59]. It can be seen that these terms oscillate at the frequency difference \( \Delta = \omega_{12} - \omega_{11} \), thus when \( \Delta \) is large enough, they may drop. This is the case treated in Ref. [58]. The present discussion is based on a situation where \( \omega_{ij} = \omega_{ij} \), and those nonsecular terms must be retained. Moreover, the presence of squeezing and the fact that \( \langle aa\rangle \neq 0 \) introduce the additional damping constants \( \eta_{ij} \) which oscillate at \( 2\omega = \omega_{11} - \omega_{12} \). Note that these terms disappear in a ladder configuration (see Ref. [54]) and in a V-type atomic configuration when \( \omega_{31} \gg \omega_{21} \) (see Ref. [58]). In the last case the atomic operators do not depend on correspondence between pairs of modes, which leads to the absence of phase sensitivity in population decay. However, in the V-type atomic configuration considered here, the central frequency of the squeezed vacuum is near the center of the doublet, thus \( 2\omega = \omega_{12} + \omega_{13} \), and all terms in \( \eta_{ij} \) must be retained. The main consequence of this fact is that some optical properties of a V atom with closely lying sublevels become phase dependent, as we shall show later. Finally, \( H_{ex}' \) represents the interaction between the atom and the external driving fields in the interaction picture

\[ H_{ex}' = \hbar \sum_{m=2}^{3} \Omega_m e^{-i(\omega_m - \omega_{11})t + \phi}|m\rangle\langle 1| + \text{H.c.} \]  
(2.10)

The radiative shifts (Lamb and Stark shifts) have been ignored. In addition, it can be shown [16] that

\[ \gamma_{23} = \frac{\sqrt{\gamma_1 \gamma_2}}{2} \left( \frac{\mu_{13}^* \mu_{12}}{|\mu_{12}|^2} \right). \]  
(2.11)

The quantum interference is maximum if the transition moment \( \mu_{13} \) is parallel to \( \mu_{12} \), but it disappears if they are perpendicular. Now we eliminate the explicit temporal dependence of the density-matrix equation through an appropriate unitary transformation \( \rho = U_N \rho U_N^\dagger, \) where \( U_N = \exp(i\Delta_3|3\rangle\langle 3|) \), \( \Delta_3 = (\omega_{31} - \omega_{11}) \) being the probe laser detuning from the resonance with the state \(|3\rangle\). In this frame, the density-matrix equation of the system can be written as

\[
\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_{ex}, \rho] - \frac{i}{\hbar} [H_0, \rho] - \frac{1}{2} \sum_{i,j=2}^{3} N(\omega_{ij}) \gamma_{ij} \left( (S_i^+ S_j^- \rho - S_j^+ S_i^-)^* + (\rho S_i^+ S_j^- - S_j^+ S_i^-) \right) - \frac{1}{2} \sum_{i,j=2}^{3} M(\omega_{ij}) \eta_{ij} \left( (S_i^+ S_j^+ - S_j^+ S_i^+)^* \rho \right) + (S_i^+ \rho S_j^+ - \rho S_j^+ S_i^+) e^{2i\phi} - \frac{1}{2} \sum_{i,j=2}^{3} \hbar M(\omega_{ij}) \eta_{ij} (S_j^+ \rho S_i^- - S_i^- S_j^+ \rho) + (S_i^- \rho S_j^- - \rho S_j^- S_i^-) e^{-2i\phi},
\]

where the Hamiltonians \( H_{ex} \) and \( H_0 \) are

\[ H_{ex} = -\hbar \sum_{m=2}^{3} \Omega_m |m\rangle\langle 1| - \text{H.c.}, \]
\[ H_0 = \hbar \Delta_3 |3\rangle\langle 3|. \]  
(2.13)

(2.14)

We assume that the frequency of the external field, \( \omega_{ex} \), lies in the middle of the excited sublevels, \( \omega \), and \( N(\omega_{31}) = N(\omega_{21}) = 2 \), and \( M(\omega_{31}) = M(\omega_{21}) = M \). Then, the evolution equations of the density-matrix elements in the rotating frame take the form

\[ \frac{\partial \rho_{33}}{\partial t} = -(N+1)\alpha \rho_{33} - (N+1) \rho (\rho_{33} + \rho_{32}) + \frac{N}{\alpha} \rho_{11} + i x_2 (\rho_{13} - \rho_{33}), \]
\[ \frac{\partial \rho_{22}}{\partial t} = -2(N+1)\alpha \rho_{22} - (N+1) \rho (\rho_{32} + \rho_{23}) + 2N \alpha \rho_{11} + i x_2 (\rho_{12} - \rho_{22}), \]
\[ \frac{\partial \rho_{31}}{\partial t} = -F_{31} \rho_{31} - (N+1) \rho (\rho_{31} + \rho_{32}) - \frac{2}{\alpha} |M| e^{i\phi} \rho_{13} + 2 |M| e^{i\phi} \rho_{12} - i x_2 (2 \rho_{33} + \rho_{22} - 1) - i x_2 \rho_{32}, \]
\[ \frac{\partial \rho_{21}}{\partial \tau} = -F_{21} \rho_{21} - (N+1) p \rho_{31} - 2M|e^{i\Phi}\alpha \rho_{12} - 2M|p e^{i\Phi}\rho_{13} - i x_3 (\rho_{23} + 2 \rho_{22} - 1) - i x_3 \rho_{23}, \]
\[ \frac{\partial \rho_{23}}{\partial \tau} = -F_{23} \rho_{23} - (N+1) p (\rho_{22} + \rho_{33}) + 2Np\rho_{11} + i x_2 \rho_{13} - i x_3 \rho_{23}, \]
where we have introduced a dimensionless time \( \tau \), dimensionless variables
\[ \tau = \frac{\sqrt{\gamma_2 \gamma_3}}{2} \tau, \]
\[ \alpha = \sqrt{\frac{\gamma_2}{\gamma_3}}, \]
\[ x_j = \frac{2}{\sqrt{\gamma_2 \gamma_3}} \Omega_j, \]
\[ p = \frac{2}{\sqrt{\gamma_2 \gamma_3}} \gamma_2, \]
\[ F_{31} = \left[ (N+1) \frac{1}{\alpha} + N \left( \alpha + \frac{1}{\alpha} \right) + i \frac{2}{\sqrt{\gamma_2 \gamma_3}} \Delta_3 \right], \]
\[ F_{21} = \left[ (N+1) \alpha + N \left( \alpha + \frac{1}{\alpha} \right) \right], \]
\[ F_{23} = \left[ (N+1) \left( \alpha + \frac{1}{\alpha} \right) - i \frac{2}{\sqrt{\gamma_2 \gamma_3}} \Delta_3 \right], \]
and the relative phase
\[ \Phi = 2\phi - \phi_e, \]
which represents the phase difference between the two coherent fields and the squeezed field. The main difference between our equations and those studied by Paspalakis and Knight [26] are the terms proportional to \( N \) and \( M \), which account for the presence of the squeezed vacuum field. In fact, if \( N=M=0 \), Eqs. (2.15) reduce to those obtained by Paspalakis et al. [26,27]. From Eqs. (2.15) we can see that the system is sensitive to the amplitude and the phase of the squeezed field through the terms depending of \( N \) and \( M \). The \( M \) terms introduce a phase dependence on the relaxation processes in the polarization due to the coupling of \( \rho_{31} \) term with \( \rho_{13} \) and \( \rho_{12} \).

III. STEADY-STATE POPULATIONS AND GAIN LINE SHAPES

We proceed to analyze the influence of the squeezed vacuum on the steady-state behavior of the system by setting \( \partial \rho_{ij}/\partial t = 0 \), so that Eqs. (2.15) reduce to a set of algebraic equations. A single inspection of these equations reveals that the atomic system becomes sensitive to the relative phase between the coherent fields and the squeezed vacuum, \( \Phi \). In the following, we shall analyze the dynamics as a function of the probe laser detuning, \( \Delta_3 \). Since the frequency of both the pump and probe fields is equal, tuning the probe laser means changing the separation between the excited levels (\( \Delta_3 = \omega_{32} \)). We are interested in the steady-state behavior of the probe transition \([3] \rightarrow [1]\), that is, in the population difference \( \rho_{33} - \rho_{11} \), and in the probe line shape of the system. The probe absorption is computed as follows [38]: the polarization \( \rho_{31} \) can be expanded in powers of the weak field \( x_3 \) as
\[ \rho_{31} = \rho_{31}^{(0)} (\Delta_2 , \Delta_3 , x_2 ) + \rho_{31}^{(1)} x_3 + O(x_3^2) + \cdots, \]
where \( \rho_{31}^{(0)} = \rho_{31}(x_3 = 0) \) is the lowest-order nonlinear susceptibility (see Eq. (7) in Ref. [38]). Then, the linear probe absorption is related with the imaginary part of \( \rho_{31}^{(1)} = (\rho_{31} - \rho_{31}^{(0)})/x_3 \). In this way, a negative value of \( \text{Im}(\rho_{31}^{(1)}) \) means that the system exhibits gain.

The present physical situation contains a large number of parameters. In order to show the influence of the squeezed vacuum we shall restrict the range of variation of the parameters involved. First of all we will consider the case in which the spontaneous decay rates are different \( (\gamma_2 > \gamma_3) \). The reason for this election is based on both theoretical and experimental considerations. It has been shown by Zhu [62,63] and Menon and Agarwal [55] that gain without inversion in a V-type atom requires that the decay rate from atomic level \( 2 \) to \( 1 \) exceed that from state \( 3 \) to \( 1 \). In experimental situations with alkali V-type atoms, the ratio between the spontaneous decay rates are greater than unity. For example, in the \(^{85}\text{Rb} \) atomic vapor, \( \gamma_2 = 4 \text{ MHz} \) and \( \gamma_2 = 20 \text{ MHz} \) \((\gamma_2/\gamma_3 = 5) \) [64]. On the other hand, typical values of the Rabi frequencies used in experiments range from 0 to 240 MHz, i.e., \( \Omega_2 \in [0.60 \gamma_3] \) [64]. In our calculations we take \( \Omega_2 = 20 \gamma_3 \), \( \Omega_3 = 1 \gamma_3 \), and \( \gamma_2 = 6 \gamma_3 \). The election of these particular values will allow us to compare our numerical results with those reported in previous relevant works in V-type atoms with quantum interference but with standard vacuum [26,55,62,63]. We would like to point out that very similar results are obtained for other values of the mentioned parameters.

For the sake of completeness, first we analyze the behavior of the system damped by the standard vacuum, i.e., we set \( N=M=0 \). Population inversion and probe line shapes are given in Figs. 2(a) and 2(b), respectively, versus the probe detuning in the presence of quantum interference \( (\rho = 0.98) \), showing the well-known double-picked structure located at \( \Delta_3 = \pm \Omega_3 = \pm \Omega_{20} \sqrt{1 - p^2} \), thus gain with population inversion is obtained. This situation corresponds with transitions between state \( 3 \) and the Autler-Townes doublet (dressed states) generated by the pump field. We remind here that quantum interference is essential to produce the gain, which is in agreement with previous results [38]. Note also that due to the field polarization arrangement, the strength of quantum interference must be smaller than 1 \((p < 1) \).

Let us study the influence of the squeezed vacuum on the scenario described above. We assume a perfect squeezing condition, i.e., \( |M| = \frac{1}{\sqrt{N(N+1)}} \), and \( \Phi = \pi \). For a small range of values of the squeezed phase around \( \pi \) the most
interesting results are obtained (see below in this section for more details). A drastically different behavior is found in the presence of the squeezed vacuum with regard to that shown in Figs. 2~\textit{a}~and 2~\textit{b}~. In this case, gain \textit{without} population inversion is obtained at the Autler-Townes doublet \cite{see Figs. 2~\textit{c}~and 2~\textit{f}~}. Note that this behavior is obtained when the atom is damped by a thermal field \cite{Figs. 2~\textit{c}~and 2~\textit{d}~} and for the case of a squeezing field \cite{Figs. 2~\textit{e}~and 2~\textit{f}~}. The main differences between these two situations arise at the central region of the spectrum where gain is obtained over the whole range of detunings in the case with $\mathcal{M} \neq 0$. Note further that the peak values at the doublet are higher when $\mathcal{M} > 0$ than in the case with $\mathcal{M} = 0$. The origin of inversionless gain at the Autler-Townes doublet can be explained in certain cases, in terms of population inversion in the dressed states. It is well known that the effect of the strong field alone is to produce dressed states \cite{[60]} (see the following section for more details). In order to analyze the combined effect of VIC and the squeezing field, we carried out a calculation for determining a state diagram in the $(p, N)$ plane. This will provide us with a map in the parameter space delimiting the regions where the system exhibits gain or absorption. Figure 3 presents the results obtained for the above-mentioned parameters. Note that for large values of the quantum interference strength and low values of $N$, gain with inversion is obtained. By increasing the value of $N$, the system presents gain without inversion in the bare basis, in agreement with Figs. 2~\textit{e}~and 2~\textit{f}~. The threshold value of $N$ at which the system exhibits inversionless gain depends on the degree of quantum interference. Note also that for low values of $\mathcal{P}$, the effect of increasing $N$ is to switch the system from gain to absorption. The most remarkable feature is that found for moderate values of $\mathcal{P}$, where the effect of the squeezing field is to produce gain without population inversion, neither in the bare nor in the dressed basis.

A significant effect takes place if we consider the case with approximately equal decay rates $\gamma_3 = \gamma_2$. In this regime and with standard vacuum, Menon and Agarwal \cite{[55]}, and Ficek and Swain \cite{[31]} pointed out that quantum interference

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(a),(c),(e) Population difference $\rho_{33} - \rho_{11}$ and (b),(d),(f) imaginary part of the susceptibility $\rho_{31}^{(1)}$ vs the dimensionless probe detuning $\Delta_3 / \gamma_3$. The parameters are $\gamma_2 = 6 \gamma_3$, $\Omega_20 = 20 \gamma_3$, $\Omega_30 = \gamma_3$, and $p = 0.98$. We consider the standard vacuum, i.e., $N = \mathcal{M} = 0$, in panels (a),(b). In the rest of the panels the squeezed parameter $N$ is 1. In panels (c),(d) the squeezed parameter $\mathcal{M}$ is zero, while in panels (e),(f) the squeezed parameter is $|\mathcal{M}| = \sqrt{N(N+1)}$ and $\Phi = \pi$.}
\end{figure}
is not able to induce gain in the system. Specifically, they showed that this gain occurs only if the decay rate of the probe transition is lower than the decay rate of the driving transition. In particular, at $p = 1$ they found the following condition $\gamma_2 < \gamma_3 / 2$. However, we have found that this restriction can be surpassed when the atom is damped by the squeezed vacuum. The probe gain (or absorption) is plotted in Fig. 4(a) at different values of the squeezed parameter $N$. At low values of $N$ (dotted-dashed line) $\Im(\rho_{31}^{(1)})$ shows two absorption peaks located at $\Delta_3 = \pm \Omega_2 = \pm \Omega_2 \sqrt{1-p^2}$, corresponding to the transitions between state $|3\rangle$ and the Autler-Townes doublet (dressed states). As $N$ increases, the first feature found is the splitting of the absorption peaks (solid line). By further increasing $N$, the absorption curve shows a dispersionlike behavior at the Autler-Townes (dotted line), and finally, at high values of $N$ (dashed line), we find that both peaks reach negative values, which means that the system exhibits gain. Concerning population difference $\rho_{31} - \rho_{11}$, we find that no inversion is obtained in all cases. In conclusion, by increasing the strength of the squeezed vacuum, the system can switch from absorption to gain without population inversion. The threshold value of $N$ that separates the absorption and gain regimes depends on the value of the quantum interference parameter. We plot in Fig. 4(b) a state diagram in the plane $(p, N)$ which shows the two different regimes, absorption and inversionless gain. We must point out that in this case of equal decay rates ($\gamma_2 = \gamma_3$), the origin of inversionless gain at the Autler-Townes doublet can be explained in terms of population inversion in dressed states (see the following section). The inspection of the state diagram shown in Fig. 4(b) reveals that we can lead the system from absorption to gain by changing the quantum interference parameter $p$.

We conclude this section by analyzing the influence of the relative phase between the coherent fields and the squeezed field in the dynamics of the V-type atom. We obtain that for $\Phi = 0$ no gain is attained in any of the cases considered. To emphasize the effect of the relative phase on the probe gain without inversion, we depict in Fig. 5 the probe gain (solid line) and the inversion (dashed line) when the relative phase between the squeezed vacuum and the coherent fields is varied. It can be seen that both the inversion and the probe gain are periodical functions of the relative phase $\Phi$, thus by changing this phase, the probe laser may experience gain, transparency, or absorption. It is worth noting that there are small regions close to $\Phi = \pi$ where both the gain and the inversion take simultaneously negative values. Therefore this relative phase plays an important role in the behavior of the system allowing the atom to exhibit gain without population inversion.

FIG. 3. State diagram in the plane $(p,N)$ showing the different dynamical regimes. Other parameters are $\gamma_2 = 6 \gamma_3$, $\Omega_{20} = 20 \gamma_3$, $\Omega_{30} = \gamma_3$, and $\Phi = \pi$.

FIG. 4. (a) Imaginary part of the susceptibility $\rho_{31}^{(1)}$ vs the dimensionless probe detuning $\Delta_3 / \gamma_3$ at different squeezed parameters; $\Phi = \pi$, $N = 0.1$ (dotted-dashed line), $N = 0.3$ (solid line), $N = 0.6$ (dotted line), and $N = 1.2$ (dashed line). The rest of the parameters are $\gamma_2 = \gamma_3$, $\Omega_{20} = 20 \gamma_3$, $\Omega_{30} = \gamma_3$, and $p = 0.98$. (b) State diagram in the plane $(p,N)$ showing the different dynamical regimes.

IV. DRESSED-STATE ANALYSIS

As described in the preceding section, V-type atoms exhibit inversionless gain due to the interplay between quantum interference and squeezed vacuum. The physics associated with the properties of the probe transition can be further explored by working in the basis of quantum dressed states.
The corresponding eigenvalues are given by photons in the laser mode. The dressed states of the system of the interaction Hamiltonian, Eq. (2.13) with \( m = 2 \). In this situation, the eigenstates of the interaction Hamiltonian are given by

\[
|\pm,\nu_{\omega}\rangle = \frac{1}{\sqrt{2}} |1,\nu_{\omega}\rangle \pm \frac{1}{\sqrt{2}} |2,\nu_{\omega} - 1\rangle,
\]

\[
|\pm,\nu_{\omega}\rangle = \frac{1}{\sqrt{2}} |1,\nu_{\omega}\rangle \pm \frac{1}{\sqrt{2}} |2,\nu_{\omega} - 1\rangle, \quad (4.1)
\]

\[
|3,\nu_{\omega}\rangle = |3,\nu_{\omega} - 1\rangle.
\]

The corresponding eigenvalues are given by \( E_{\pm} = \hbar \nu_{\omega} (\omega_{21} \pm \Omega_2) \) and \( E_{\pm} = \hbar \nu_{\omega} (\omega_{21} + \omega_{32}) \). \( \nu_{\omega} \) being the number of photons in the laser mode. The dressed states \(|\pm,\nu_{\omega}\rangle\), \(|\mp,\nu_{\omega}\rangle\), and \(|3,\nu_{\omega}\rangle\) form an infinite ladder of three-state manifolds. Adjacent manifolds are separated by \( \omega_{21} \). Inside a manifold the states \(|+\nu_{\omega}\rangle\) and \(|-\nu_{\omega}\rangle\) are separated from the state \(|3,\nu_{\omega}\rangle\) by \( 2\Omega_2 - \omega_{32} \) and \( 2\Omega_2 + \omega_{32} \), respectively [see Fig. 6(a)]. With the dressed states of the driven system, we may easily predict transition frequencies and calculate transition dipole moments and spontaneous emission rates between dressed states of the system. It is easily verified that nonzero dipole moments occur only between dressed states within neighboring manifolds. Using Eq. (4.1) we find that the transition dipole moments between \(|i,\nu_{\omega}\rangle\) and \(|j,\nu_{\omega} - 1\rangle\) \((i,j = +, -, 3)\) are

\[
\mu_{+,\nu_{\omega}:+,\nu_{\omega} - 1} = \frac{1}{2}\mu_{12},
\]

\[
\mu_{+,\nu_{\omega}:+,\nu_{\omega} - 1} = \frac{1}{2}\mu_{12},
\]

\[
\mu_{+,\nu_{\omega}:+,\nu_{\omega} - 1} = 0,
\]

\[
\mu_{-,\nu_{\omega}:-,\nu_{\omega} - 1} = \frac{1}{2}\mu_{12},
\]

\[
\mu_{-,\nu_{\omega}:-,\nu_{\omega} - 1} = \frac{1}{2}\mu_{12},
\]

\[
\mu_{-,\nu_{\omega}:-,\nu_{\omega} - 1} = 0,
\]

\[
\mu_{3,\nu_{\omega}:+,\nu_{\omega} - 1} = \frac{1}{\sqrt{2}}\mu_{13},
\]

\[
\mu_{3,\nu_{\omega}:+,\nu_{\omega} - 1} = \frac{1}{\sqrt{2}}\mu_{13},
\]

\[
\mu_{3,\nu_{\omega}:+,\nu_{\omega} - 1} = 0, \quad (4.2)
\]

FIG. 6. (a) Diagram of transitions among the dressed states. (b) By changing the splitting the state \(|3,\nu_{\omega}\rangle\) is tuned to the state \(|+,\nu_{\omega}\rangle\): relevant transitions (those preserving the transition frequency) are indicated.
where \( \tilde{\mu}_{i,N_w} = \langle i,N_w | \mu | j,N_w - 1 \rangle \) and \( \tilde{\mu} \) is the total dipole moment of the atom. In Fig. 6(a) we present the dressed states of two neighboring manifolds and the possible transitions among them.

The probe absorption or gain at the Autler-Townes doublet can be explained by analyzing the population in the dressed states. Let us study the gain at the positive component of the Autler-Townes doublet, which occurs at \( \Delta_3 = 2\Omega_2 \). In this case the level \( |3,N_w\rangle \) is degenerate with the state \( |+,N_w\rangle \) as shown in Fig. 6(b). Consequently, the resonant transition at the frequency of the probe field arises from the transition between the states \( |3,N_w\rangle \) and \( |+,N_w-1\rangle \). Applying a similar reasoning, the negative component which occurs at \( \Delta_3 = -2\Omega_2 \) corresponds to the transition \( |3,N_w\rangle \rightarrow |-,N_w-1\rangle \). The population of the dressed states are plotted in Fig. 7(a) with the same set of parameters as in Figs. 2(e) and 2(f). It can be seen that the system exhibits inversion in the dressed-state basis, i.e., \( \rho_{33} - \rho_{++} > 0 \) at \( \Delta_3 = +2\Omega_2 \) and \( \rho_{33} - \rho_{--} > 0 \) at \( \Delta_3 = -2\Omega_2 \). Therefore, the probe gain found in the Autler-Townes doublet can be attributed to inversion in the dressed states. On the other hand, as we showed in Fig. 2(f), the probe gain was also obtained in the central region of the spectrum. This fact cannot be explained with the previous argument. In order to bring some light to this phenomenon, we write the steady state of \( \text{Im}(\rho_{31}^{(1)}) \) as a function of the rest of the variables [from Eqs. (2.15)] for the case with \( \Phi = \pi \), which reads

\[
\text{Im}(\rho_{31}^{(1)}) = \frac{1}{(N+1)^2 + (2\Delta_3 / \gamma_3)^2 + 2(N+1)|M|^2}\left[\alpha(N+1) \times (\rho_{11} - \rho_{33}) - x_2 \alpha(N+1) \Re(\rho_{32}^{(1)}) - \rho \alpha(N+1) \times (N+1 + 2|M|) \Im(\rho_{21}^{(1)}) - 2x_2 \alpha(\Delta_3 / \gamma_3) \Im(\rho_{32}^{(1)}) + 2\rho \alpha(N+1 - 2|M|) \times (\Delta_3 / \gamma_3) \Re(\rho_{21}^{(1)}) - 4|M|[(\Delta_3 / \gamma_3) \Re(\rho_{31}^{(1)}) - 4|M|[(\Delta_3 / \gamma_3) \Re(\rho_{31}^{(1)})] \right].
\]  

(4.3)

It can be seen from the above formula that the probe polarization depends on the real and imaginary parts of the coherences \( \rho_{21}^{(1)} \) and \( \rho_{32}^{(1)} \), the real part of probe coherence \( \rho_{31}^{(1)} \), and the population difference \( \rho_{11} - \rho_{33} \). Note that in the absence of population inversion, i.e., \( \rho_{33} - \rho_{11} < 0 \), the contribution to \( \text{Im}(\rho_{31}^{(1)}) \) of the first term is always positive, indicating absorption. Therefore, in this case, the gain should be attributed to the rest of the terms. To appreciate how the probe gain is contributed by the different terms, we have plotted in Fig. 7(b) the individual contributions to \( \text{Im}(\rho_{31}^{(1)}) \) as a function of the probe detuning using the same parameters as in Fig. 2(e) and 2(f). Let us focus on the case of probe resonance, i.e., \( \Delta_3 = 0 \). In this case, we can see from Eq. (4.3) that the last three terms vanish, and Eq. (4.3) reduces to

\[
\text{Im}(\rho_{31}^{(1)}) \times (\rho_{11} - \rho_{33}) - x_2 \Re(\rho_{32}^{(1)}) - p(N+1+2|M|) \times \text{Im}(\rho_{21}^{(1)}).
\]  

(4.4)

FIG. 7. (a) Population of dressed states vs the dimensionless probe detuning \( \Delta_3 / \gamma_3 \). (b) Different terms of the probe gain given by Eq. (4.3) which are proportional to the different variables: \( \rho_{11} - \rho_{33} \) (dotted line), \( \Re(\rho_{32}^{(1)}) \) (dashed line), \( \text{Im}(\rho_{31}^{(1)}) \) (solid line), \( \Re(\rho_{21}^{(1)}) \) (dotted-dashed line), \( \Re(\rho_{31}^{(1)}) \) (short dotted line), and \( \text{Im}(\rho_{32}^{(1)}) \) (dotted-dotted-dashed line). The parameters are \( \gamma_2 = 6 \gamma_3, \Omega_2 = 20 \gamma_3, \Omega_3 = \gamma_3, p = 0.98 \) and the squeezed parameters \( N = 1 \) and \( \Phi = \pi \). This set of parameters corresponds to Figs. 2(e) and 2(f).

The first two terms are the conventional contributions that appear in V-type atoms with well-separated upper levels \([62,63]\) whereas the last term is crucially dependent on the \( p \) parameter and the squeezing parameters. This new term plays a fundamental role in obtaining gain at the center line. When the atom is damped by the standard vacuum (\( N = 0 \) [see Fig. 2(b)], or a thermal field (\( M = 0 \) [see Fig. 2(d)]), the contribution of this term is not able to produce net gain. Therefore, the presence of squeezed vacuum is essential to produce inversionless gain at the center line [see Fig. 2(f)].
It should be noted that for a set of parameters inside the layered region in Fig. 3 the probe gain present at the Autler-Townes doublet cannot be attributed to the population inversion in the dressed-state basis. A particular example of this behavior is depicted in Fig. 8(a) where populations of the dressed states are shown. Note that \( \rho_{33} - \rho_{++} \) and \( \rho_{33} - \rho_{--} \) take negative values in all spectral range, that is, the system presents gain without inversion in any standard state basis. Figure 8(b) presents the individual contributions to \( \text{Im}(\rho_{11}^{(1)}) \) as a function of the probe detuning by using Eq. (4.3). The highest contribution to the gain at the Autler-Townes doublet arises from the two-photon coherence term \( \text{Re}(\rho_{32}^{(1)}) \), as in the previous case. However, we must point out that the contribution of the coherence term \( \text{Im}(\rho_{21}^{(1)}) \) is essential in order to achieve gain in the system. Therefore, as in the previous case, we found that the combination of these two terms is responsible for the gain behavior.

The origin of gain in the positive component of the Autler-Townes doublet can be explained by carrying out the analysis in the dressed basis: the principal steps of the calculations are indicated in the Appendix. Specifically, we consider the case with equal decay rates and we take a value of the squeezed parameter \( N \) increases [see Fig. 4(a)]. We plot in Fig. 9(a) the populations \( \rho_{33} \) and \( \rho_{++} \) versus the squeezed parameter \( N \) for the same parameters as Fig. 4(a). Effectively, we find that at high values of \( N \) there is no population inversion in the bare states, whereas population inversion is achieved in the dressed states (\( \rho_{33} > \rho_{++} \)). Therefore, the gain found in these cases is associated with population inversion in the dressed states. The threshold value for the squeezed parameter, \( N_{\text{th}} \), above which this population inversion appears, can be estimated by imposing the condition \( \rho_{33} = \rho_{++} \) in the steady-state solution. This equation must be solved numerically, predicts a value of \( N_{\text{th}} = 0.68 \) which is in accordance with the results presented in Fig. 4. We must point out that a thermal field will not produce gain in the same circumstances. This fact can also be proved by analyzing the dependence of \( \rho_{++} \) and \( \rho_{33} \) versus \( N \), and effectively, as it can be seen in Fig. 9(b), there is no crossing between both curves.

A similar analysis could be carried out to explain the physical origin of gain in the negative part of the curves. In this case, the gain is associated with a population inversion in the dressed basis, \( \rho_{33} > \rho_{--} \). Note that in this case the relevant transition is \([3,N_u] \rightarrow [-,N_u - 1]\) as given in Eq. (4.2).

**V. EFFECTS OF THE SQUEEZED VACUUM ON THE REFRACTION INDEX**

It is wellknown that the vacuum-induced coherence can lead to a high index of refraction with vanishing absorption [12]. Akram et al. have shown that a large index of refraction without absorption can be achieved in a two-level atom driven by a squeezed vacuum field [65]. In this section, we analyze the combined effect of VIC and the squeezing field in a V-type atom. Figures 10(a)–10(c) depict \( \text{Im}(\rho_{11}^{(1)}) \) and \( \text{Re}(\rho_{32}^{(1)}) \) versus the probe detuning whether the squeezed field is in action or not. First, let us consider the case where the atom is damped by the standard vacuum (\( N = 0 \)). In the absence of quantum interference (\( \rho = 0 \)), a high refractive index is always accompanied by nonzero absorption [point A in Fig. 10(a)]. In the presence of quantum interference, the maximum value of the refraction index is achieved with gain [point B in Fig. 10(b)]. Finally, when both VIC and the
squeezing field act simultaneously, there is a probe detuning at which the probe laser experiences both vanishing absorption and a high refraction index [point C Fig. 10(c)]. The peak value of the refraction index depends on the value of $N$.

There is another remarkable phenomenon which is shown in the panel of Fig. 11 where we present the dramatic influence of the squeezed field in the refraction index. For suitable values of the squeezed parameters, we found a nearly zero plateau in both the absorption and dispersion curves in a wide frequency region between the Autler-Townes doublet. This phenomenon is, of course, induced by the coherence created by spontaneous emission interference and the squeezed field. In addition, this region of very reduced absorption and nearly unity refractive index can be enlarged by increasing the driving field [see Figs. 11(b) and 11(c)]. This is of interest when analyzing the propagation of pulses without appreciable attenuation and distortion.

Finally, we would like to point out that when the pump field is detuned from the atomic resonance one of the Autler-Townes doublet components flips sign providing significant absorption, while the other exhibits gain with population inversion in the dressed basis. Simultaneously, a higher index of refraction with vanishing absorption is obtained.

VI. CONCLUSIONS

In this paper we have discussed the combined effect of the quantum interference and the squeezed field in a V-type atom with nearly degenerated excited levels, driven by a strong coherent field. We have analyzed the degenerate pump-probe spectroscopy and steady-state populations, provided the en-
energy separation between the two upper levels can be scanned. Inversionless gain induced by the squeezed field can be obtained for a wide range of values for both the squeezed field parameters and the probe detuning. We note that in the present configuration, inversionless gain is achieved in the absence of incoherent pumping, in contrast with previous works [26,63]. We have shown that in the case of approximately equal decays, the probe experiences gain when the atom is damped by the squeezed field, thus surpassing the restriction \( \gamma_2 > 2 \gamma_1 \) predicted by Menon and Agarwal [55] and explained by Ficek and Swain [33]. We have demonstrated that the inversion and the gain may be controlled by varying the relative phases between the coherent fields and the squeezing vacuum field. In certain cases, the origin of this inversionless gain can be explained by means of population inversion in the dressed states. It should be noted that for adequate values of \( p \) and \( N \), gain without inversion in any standard atomic basis is obtained. Furthermore, we have found that the squeezed field induces a notable influence on the refraction index. For suitable values of the squeezed field parameters we found a nearly zero plateau in both the absorption and dispersion curves, which allows the propagation of nearly dispersionless and distortionless pulses.

To conclude, we discuss some conditions required to prove experimentally the theoretical results predicted in this work. One is concerned with the requirement that the squeezed field modes must occupy the whole \( \Delta \) solid angle of space. This obstacle should be probably avoided by using some sort of cavity system [66] or by using a weak-amplitude fluctuating field which mimics the squeezed vacuum [67]. Furthermore, in our calculations we have used squeezing photon numbers \( N \) which are now achievable in laboratories [68]. On the other hand, quantum interference between the two decay channels may only occur when the transitions involved are nonorthogonal: as we have mentioned in the Introduction, several methods to bypass this stringent condition have been proposed. Finally, we would like to point out that the inversionless gain without incoherent pumping obtained in this work opens the possibility to achieve a reduction of the intensity fluctuations on the laser without inversion, as it happens in the squeezed-reservoir two-level lasers [69].

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APPENDIX

Some insight of the numerical results showed in Sec. IV can be gained if we perform the analysis in the dressed basis. In order to do that, we will restrict our analysis to the simple case in which the energy diagram is that shown in Fig. 6(b), where the relevant transitions are indicated: we can ignore the nonpreserving energy transitions. A lengthy but straightforward calculation allows us to obtain the equations for the population and coherences which read

\[
\frac{d \rho_{33}}{d \tau} = \Gamma_{33}^3 \rho_{33} + \Gamma_{33}^3 \rho_{33}^+ + \Gamma_{33}^3 \rho_3 + \Gamma_{33}^3 \rho_3^+ + \Gamma_{33}^3 \rho_0^+ + \Gamma_{33}^3 \rho_0^+
\]

(A1)

\[
\frac{d \rho_{33}^+}{d \tau} = \Gamma_{33}^3 \rho_{33}^+ + \Gamma_{33}^3 \rho_3^+ + \Gamma_{33}^3 \rho_3 + \Gamma_{33}^3 \rho_3^+ + \Gamma_{33}^3 \rho_0^+ + \Gamma_{33}^3 \rho_0^+
\]

(A2)

\[
\frac{d \rho_{33}}{d \tau} = \Gamma_{33}^3 \rho_{33} + \Gamma_{33}^3 \rho_{33}^+ + \Gamma_{33}^3 \rho_3 + \Gamma_{33}^3 \rho_3^+ + \Gamma_{33}^3 \rho_0^+ + \Gamma_{33}^3 \rho_0^+
\]

(A3)
where we have obtained explicit expressions of transition rates, showing how they depend on squeezing vacuum and quantum interference parameters, which are given by

\[ \Gamma_{33} = -2 \frac{N+1}{\alpha} - 2 \frac{N}{\alpha^2} \]

\[ \Gamma_{3}^{+3} = 0, \]

\[ \Gamma_{3}^{3+} = \frac{1}{\sqrt{2}} (N+1)p, \]

\[ \Gamma_{3}^{33} = \frac{1}{\sqrt{2}} (N+1)p, \]

\[ \Gamma_{0} = \frac{N}{\alpha}, \]

\[ \Gamma_{3}^{3+} = - \frac{N+1}{2} \frac{\alpha}{\alpha} + \frac{N}{2} - |M| \alpha \cos(\Phi), \]

\[ \Gamma_{3}^{3+} = -N \alpha - (N+1) \frac{\alpha}{2} - 2 |M| \alpha \cos(\Phi), \]

\[ \Gamma_{3}^{++} = \frac{1}{\sqrt{2}} |M| p e^{-i\Phi}, \]  

(A4)

\[ \Gamma_{3}^{++} = \frac{1}{\sqrt{2}} |M| p e^{i\Phi}, \]

\[ \Gamma_{3}^{3} = \frac{(N+1)\alpha}{2} + \frac{N}{2} + |M| \alpha \cos(\Phi), \]

\[ \Gamma_{3}^{3} = \frac{(N+1)\alpha}{\sqrt{2}} + \frac{Np}{\sqrt{2}} + \frac{1}{\sqrt{2}} |M| p e^{i\Phi}, \]

Note that Eqs. (A1)-(A3) are not the usual rate equations because the diagonal elements are coupled with off-diagonal elements. A single inspection of Eqs. (A1)-(A3) and (A4) reveals that populations and coherences depend on the squeezing parameters \( N, |M|, \) and \( \Phi \). Furthermore, it is to be noted that the presence of quantum interference is essential to produce the coupling between the \( |3,N_m\rangle \) and \( |+,-N_m-1\rangle \): in the case with \( p=0 \), we have \( \Gamma_{3}^{3+} = \Gamma_{3}^{++} = 0 \), thus \( \rho_{3+} \) is decoupled from \( \rho_{33} \) and \( \rho_{++} \) as it is verified from Eq. (A4). By defining the vector \( U = (\rho_{33}, \rho_{++}, \rho_{3+}, \rho_{3+})^t \), where \( t \) stands for transpose, Eqs. (A1)-(A3) can be written in a more compact form as

\[ \frac{d U}{d \tau} = KU + B, \]  

(A5)

\( K \) being the matrix of damping coefficients and \( B \) is the vector defined by \( B = (\Gamma_{33}^{33}, \Gamma_{0}^{3+}, \Gamma_{3}^{3+}, \Gamma_{0}^{3+})^t \). By setting \( dU/d\tau = 0 \), we obtained analytical expressions for steady-state population and coherences. These expressions are highly complicated and are not reproduced here for brevity.