Optical bistability using quantum interference in $V$-type atoms

M A Antón and Oscar G Calderón

Escuela Universitaria de Óptica, Universidad Complutense de Madrid, C/ Arcos de Jalón s/n, 28037 Madrid, Spain

E-mail: Antonm@eucmax.sim.ucm.es and oscargc@eucmax.sim.ucm.es

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Abstract
The behaviour of a $V$-type three-level atomic system in a ring cavity driven by a coherent field is studied. We consider a $V$ configuration under conditions such that interference between decay channels is important. We find that when quantum interference is taken into account, optical bistability can be realized with a considerable decrease in the threshold intensity and the cooperative parameter. On the other hand, we also include the finite bandwidth of the driving field and study its role in the optical bistable response. It is found that at certain linewidths of the driving field optical bistability is obtained even if the system satisfies the trapping condition and the threshold intensity can be controlled. Furthermore, a change from the optical bistability due to quantum interference to the usual bistable behaviour based on saturation occurs as the driving field linewidth increases.

Keywords: Quantum interference, atomic coherences, optical bistability, three-level atom, phase fluctuations

1. Introduction
In recent years, there has been considerable interest in the quantum interference and coherence effects in a multilevel atom system induced by coherent electromagnetic fields. Many phenomena such as electromagnetically induced transparency (EIT) [1], lasing without inversion (LWI) [2], refractive index enhancement without absorption [3], giant nonlinearity [4] and spontaneous emission cancellation [5, 6] have been predicted and experimentally demonstrated and have modified the way we look at photon absorption and emission processes as well as the way we look at field propagation.

The role of atomic coherences in the context of collective phenomenon, such as optical bistability (OB), has been subject to analysis once again. In an earlier article, Walls et al [7, 8] proposed a novel scheme for optical bistability using atomic coherence effects in three-level systems. They found that in $\Lambda$-type atoms the resulting OB is due to a population trapping in a coherent superposition of the ground state sublevels [9]. This coherent superposition is not coupled to the excited level, which leads to a strong narrow non-absorption resonance in the absorption profile. The width of this dip is intensity dependent. The non-absorption resonance dip broadens and the medium becomes transparent as the intensity of the driving field increases. The main feature of this mechanism is the following: it does not require atomic saturation and optical bistability is achieved with lower laser intensities. This interesting topic, the dynamics of $\Lambda$-type atoms in ring cavities, has been studied more recently [10]. Bentley et al show the possibility of observing an electromagnetically induced transparency central peak with a subnatural width [11].

On the other hand, in $V$-type atoms a similar dip occurs although, in this case, the coherence between upper sublevels is relaxed by the spontaneous emission to the ground level [8]. Large intensities are necessary in order for quantum coherence to influence the absorption processes. This is the reason why studies in OB in such multilevel systems have been concentrated in the $\Lambda$-type atoms. However, we expect that the problem of the relaxation of the coherence between excited sublevels in $V$-type atoms can be avoided if we consider quantum interference between upper sublevels. It is well known [9, 12, 13] that if the two upper levels are very close and damped by the usual vacuum interactions, spontaneous emission cancellation can take place. Then, the two decay pathways from the excited doublet to the ground state are not independent, which offers the possibility to trap population in
the excited levels when some particular conditions hold [14]. This similarity between $V$-type systems and $\Lambda$-type systems has been previously pointed out by Imamoglu and Harris in the limit of maximum interference [2, 15].

In this paper we investigate the behaviour in a unidirectional ring cavity of a collection of $V$-type atoms by taking into account the possibility of quantum interference between the two upper sublevels. Our atomic system (see figure 1) is similar to that studied in reference [7, 8]; however, the effects of quantum interference, which are included in our model, will substantially modify the bistable response of the system. We find optical bistability in this system with a lower intensity threshold and cooperative parameter than the usual one found in $V$-type atoms without quantum interference. Therefore, $V$-type atoms give a similar bistable behaviour to the $\Lambda$-type atoms, since the quantum interference overcomes the problem of the relaxation of the coherence of the two upper sublevels via spontaneous emission. We also find that the bistable response is very sensitive to the detuning between the driven field and the atomic levels. In real experiments, the laser undergoes finite bandwidth effects. Dalton and Knight [16] have shown that the fluctuations of the pump laser can essentially influence the optical properties of the driven atomic systems. Thus, Zhou and Swain [5] as well as Sultana and Zubairy [17] have shown that a finite linewidth destroys the spectral narrowing features in $V$ atoms. However, in the context of optical bistability, Gong et al [18] showed that in three-level $\Lambda$-type atoms driven by a strong coherent field and a weak control field, optical bistability can be led via a phase-fluctuation effect of the control field. Hu et al [19] have suggested a method of achieving phase control of the fluctuation-induced bistability with two-photon resonance in such $\Lambda$ atoms. For this reason, we also analyse the effect of the finite driven laser linewidth on the bistable behaviour.

The paper is organized as follows: section 2 establishes the model, i.e. the Hamiltonian of the system and the evolution equation of the atomic operators assuming the rotating wave approximation. Section 3 deals with the Maxwell equation in a ring cavity and the influence of the quantum resonance in the bistable response of the system. The effect of the driving field linewidth on the bistable behaviour is studied in section 4, while section 5 provides brief conclusions.

2. The model and the equations of motion

We consider a $V$-type atom consisting of two upper sublevels $|1\rangle$, $|2\rangle$ coupled to a single ground level $|3\rangle$ by a single-mode laser field with amplitude $E$ and angular frequency $\omega$. The energy-level scheme is shown in figure 1.

In order to take into account the induced-coherence effects by spontaneous emission, the two upper levels $|1\rangle$ and $|2\rangle$ are coupled by the same vacuum modes to the ground level $|3\rangle$. The resonant frequencies between the upper levels $|1\rangle$, $|2\rangle$ and the ground level $|3\rangle$ are $\omega_{13}$, $\omega_{23}$, respectively. Note that $\omega_{13} - \omega_{23} = \omega_{12}$, with $\omega_{12}$ being the frequency separation of the excited levels. The Hamiltonian of the system in the rotating wave approximation is given by [20]

$$H = \hbar \sum_{m=1}^{3} \omega_{m} |m\rangle \langle m| + \hbar \sum_{k \neq k'} g_{mk} |m\rangle \langle k| + H.c.$$ 

$$+ \hbar \sum_{m=1}^{3} \sum_{kk'} g_{mk} |m\rangle \langle k| a_{k}^\dagger a_{k} + H.c.$$ 

$$-\hbar \sum_{m=1}^{3} \Omega_{m} e^{-i\omega_{m}t} |m\rangle \langle 3| - H.c.$$ 

(1)

Here, $\hbar \omega_{m}$ are the energies of the atomic levels, $a_{k}$ ($a_{k}^\dagger$) is the annihilation (creation) operator of the $k$th mode of the vacuum field with polarization $e_{k}$ ($k = 1, 2$) and angular frequency $\omega_{k}$. The parameter $g_{mk}$ is the coupling constant of the atomic transition $|m\rangle \rightarrow |3\rangle$ with the vacuum mode

$$g_{mk} = -\sqrt{\frac{\omega_{m}}{2\hbar \gamma_{m} V}} (\vec{\mu}_{3m} \cdot e_{k}),$$

(2)

where $\vec{\mu}_{3m}$ is the dipolar moment of the transition and $\Omega_{m} = \mu_{3m} E/(2\hbar)$ is the Rabi frequency of the transition $|m\rangle \rightarrow |3\rangle$.

The system is studied using the density-matrix formalism. Following the traditional approach of Weisskopf and Wigner [20–22], we obtain the master equation for the reduced density matrix $\rho$ for the atomic system

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_{c}, \rho] + \frac{1}{2} \gamma_{12} (2A_{13}\rho A_{13} - A_{13}\rho - \rho A_{13})$$

$$+ \frac{1}{2} \gamma_{12} (2A_{23}\rho A_{23} - A_{23}\rho - \rho A_{23})$$

$$+ \gamma_{12} (2A_{23}\rho A_{13} - A_{13}\rho - \rho A_{12})$$

$$+ \gamma_{12} (2A_{31}\rho A_{23} - A_{23}\rho - \rho A_{21}),$$

(3)

where $A_{ij} = |i\rangle \langle j|$, $\gamma_{m}$ is the spontaneous decay constant of the excited sublevel $m$ ($m = 1, 2$) to the ground level and $H_{c} = -\hbar \sum_{m=3}^{2} \Omega_{m} e^{-i\omega_{m}t} |m\rangle \langle 3| - H.c.$ is the Hamiltonian of the interaction of the atomic system with the external coherent field. The effect of the quantum interference is represented by the terms with the $\gamma_{12}$ coefficient which is given by [5]

$$\gamma_{12} = \frac{\sqrt{\gamma_{1}\gamma_{2}}}{2} \left( \frac{\vec{\mu}_{13} \cdot \vec{\mu}_{23}}{\mu_{13} \mu_{23}} \right).$$

(4)

The quantum interference is maximum if the transition moment $\vec{\mu}_{13}$ is parallel to $\vec{\mu}_{23}$, but it disappears if they are perpendicular. The evolution equations of the density matrix elements in the rotating frame take the form [5, 6, 14, 23]

$$\frac{\partial \rho_{11}}{\partial t} = -\gamma_{1} \rho_{11} - \gamma_{12} (\rho_{12} + \rho_{21}) + i\Omega_{1} \rho_{31} - i\Omega_{2}^* \rho_{32},$$

(5)
\[
\frac{\partial \rho_{22}}{\partial t} = -\gamma_{2} \rho_{22} - \gamma_{12} (\rho_{12} + \rho_{21}) + i\Omega_{2} \rho_{32} - i\Omega_{2}^* \rho_{23}, \quad (6)
\]
\[
\frac{\partial \rho_{13}}{\partial t} = -\Gamma_{13} \rho_{13} - \gamma_{12} \rho_{23} - i\Omega_{1} (\rho_{11} - \rho_{33}) - i\Omega_{1} \rho_{12}, \quad (7)
\]
\[
\frac{\partial \rho_{23}}{\partial t} = -\Gamma_{23} \rho_{23} - \gamma_{12} \rho_{13} + i\Omega_{2} (\rho_{33} - \rho_{22}) - i\Omega_{1} \rho_{21}, \quad (8)
\]
\[
\frac{\partial \rho_{12}}{\partial t} = -\Gamma_{12} \rho_{12} - \gamma_{12} (\rho_{11} + \rho_{22}) + i\Omega_{1} \rho_{32} - i\Omega_{1}^* \rho_{13}, \quad (9)
\]

where \(\Gamma_{13} = (\gamma_{1}/2 + i\Delta_{1}), \Gamma_{23} = (\gamma_{2}/2 + i\Delta_{2})\) and \(\Gamma_{12} = [(\gamma_{1} + \gamma_{2})/2 + i\omega_{12}]\). The laser detuning from the resonance with the state \(|m\rangle\) \((m = 1, 2)\) is \(\Delta_{m} = (\omega_{m3} - \omega)\). In order to compare the results with the two-level system and the \(\Lambda\) system [7, 8], the following normalized variables are defined:

\[
F_{ij} = \frac{2}{\sqrt{\gamma_{1} \gamma_{2}}} \Gamma_{ij}, \quad (10)
\]
\[
x_{j} = \frac{2}{\sqrt{\gamma_{1} \gamma_{2}}} \Omega_{j}, \quad (11)
\]
\[
p = \frac{2}{\sqrt{\gamma_{1} \gamma_{2}}} \gamma_{12}, \quad (12)
\]

With these definitions and introducing the normalized time \(\tau = (\sqrt{\gamma_{1} \gamma_{2}}/2)t\), the system of equations (5)–(9) reads

\[
\frac{\partial \rho_{11}}{\partial \tau} = -2 \sqrt{\frac{\gamma_{1}}{\gamma_{2}}} \rho_{11} - p(\rho_{12} + \rho_{21}) + i\xi_{1} \rho_{31} - i\xi_{1}^* \rho_{13}, \quad (13)
\]
\[
\frac{\partial \rho_{22}}{\partial \tau} = -2 \sqrt{\frac{\gamma_{2}}{\gamma_{1}}} \rho_{22} - p(\rho_{12} + \rho_{21}) + i\xi_{2} \rho_{32} - i\xi_{2}^* \rho_{23}, \quad (14)
\]
\[
\frac{\partial \rho_{13}}{\partial \tau} = -F_{13} \rho_{13} - p\rho_{23} - i\xi_{1} (2\rho_{11} + \rho_{22} - 1) - i\xi_{2} \rho_{12}, \quad (15)
\]
\[
\frac{\partial \rho_{23}}{\partial \tau} = -F_{23} \rho_{23} - p\rho_{13} + i\xi_{2} (1 - \rho_{11} - 2\rho_{22}) - i\xi_{1} \rho_{21}, \quad (16)
\]
\[
\frac{\partial \rho_{12}}{\partial \tau} = -F_{12} \rho_{12} - p(\rho_{11} + \rho_{22}) + i\xi_{1} \rho_{32} - i\xi_{2}^* \rho_{13}, \quad (17)
\]

where we have assumed that \(\rho_{11} + \rho_{22} + \rho_{33} = 1\). An inspection of equations (13)–(17) displays several terms such as \(p(\rho_{12} + \rho_{21})\), \(p\rho_{13}\) or \(p\rho_{23}\) which are not present in the usual density-matrix equations of V-type three-level systems [7, 8]. These new terms are due to the quantum interference between the two decay channels to the ground level.

In the following, we shall analyse the dynamics as a function of the dimensionless detuning \(\delta_{1} \equiv (\Delta_{1} + \Delta_{2})/\sqrt{\gamma_{1} \gamma_{2}}\), which is the laser detuning from the resonance with the centre of the two excited levels (see figure 1). In order to analyse the effect of the quantum interference on the response of the system, we have solved the equations (13)–(17). After an initial transient the system reaches a steady state. Let us focus on the steady-state regime of the system. We specifically present the absorption characteristics of the optical coherence on the transition \(|3\rangle \rightarrow |1\rangle\) and \(|3\rangle \rightarrow |2\rangle\), i.e. \(\text{Im}(\rho_{13} + \rho_{23})\), and the behaviour of the total population of the two excited sublevels, \(\rho_{11} + \rho_{22}\), as a function of the detuning \(\delta_{1}\), for the cases with and without quantum interference, i.e. \(p = 1\) and \(p = 0\), respectively. In the rest of the section we shall consider the case with \(\gamma_{1} = \gamma_{2}\) [24] and \(|\mu_{31}| = |\mu_{32}|\), so then \(x_{1} = x_{2} = x\).

![Graph](image.png)

**Figure 2.** Imaginary part of the susceptibility \(\rho_{13} + \rho_{23}\) versus the tuning \(\delta_{1}\) for a driven coherent field \(|x| = 0.3\) (solid curve), \(|x| = 0.2\) (dashed curve) and \(|x| = 0.1\) (dotted curve). The parameter \(2\omega_{12}/\sqrt{\gamma_{1} \gamma_{2}} = 0.03\). Comparison between (a) \(p = 0\) and (b) \(p = 1\).

The typical behaviour of \(\text{Im}(\rho_{13} + \rho_{23})\) is depicted in figure 2(a) for the case \(p = 0\) and in figure 2(b) for the case \(p = 1\) for different values of the driving field without cavity. In the absence of quantum interference, the absorption displays a usual Lorentzian line shape. However, if \(p = 1\), a sharp dip appears. The width of this dip is intensity dependent and broadens as the field increases. At zero detuning (\(\delta_{1} = 0\)) the absorption is zero. It is well known that this non-absorption resonance is due to a cancellation of the spontaneous emission produced by the quantum interference in the two possible decay channels to the ground sublevel [5, 6]. For our purposes, it is interesting to note that at values of \(\delta_{1}\) slightly different to zero, the atomic system absorbs for \(x = 0.1\) but it is almost transparent for \(x = 0.3\), due to the broadening of the dip. We shall point out that no saturation effects take place at this field value.

In figure 3 the population of the excited sublevels \(\rho_{11} + \rho_{22}\) is plotted as a function of the driving field \(x\), for different atomic detunings, \(\delta_{1}\). In the case of \(p = 0\) (see figure 3(a)), \(\rho_{11} + \rho_{22}\) reaches the maximum value of \(1/2\), so the population inversion is \((\rho_{11} + \rho_{22}) - \rho_{33} = 0\). In this situation the V-type atom is equivalent to a two-level atom. On the other hand,
Substituting (18) in equations (13)–(17), we have for the coherences

\[ \rho_{23} = \rho_{13} = 0, \quad \rho_{21} = \rho_{12} = -\frac{1}{2}. \]  

From (18) and (19) we see that in the steady-state limit, the atom driven by the laser field evolves into the pure state

\[ |\psi(\infty)\rangle = \frac{e^{i\phi}}{\sqrt{2}} (|1\rangle - |2\rangle). \]  

In this state, there is a strong cancellation between the atomic transitions \(|1\rangle \rightarrow |3\rangle|\) and \(|2\rangle \rightarrow |3\rangle|\), and the atom is trapped in its two upper sublevels. However, this coherent superposition is very sensitive to the detuning, \(\delta_k\), so that if \(\delta_k \neq 0\), the trapped state is destroyed and the atom absorbs. As we shall see below, this mechanism is responsible for achieving optical bistability.

3. Quantum interference effect on the steady-state response

In order to study the optical response of \(V\)-type three-level atoms in a cavity, let us consider a medium of length \(L\) composed by this type of atom immersed in a unidirectional ring cavity with total length \(L_T\) (see figure 4). The incident coherent field \(E_I\) is taken to be linearly polarized in the same direction as the transition dipole moments and propagates in the \(z\) direction. For this reason, we shall prescind from the vectorial character of the different variables. The field \(E\) is given by

\[ E = \frac{1}{2}E(z,t)e^{-i\omega t} + c.c., \]

where \(E(z,t)\) is the slowly varying field envelope. The propagation of the laser field in the medium is governed by Maxwell’s wave equation, which in the slowly varying envelope approximation reduces to

\[ \epsilon \frac{\partial E(z,t)}{\partial z} + \frac{\partial E(z,t)}{\partial t} = -\frac{\omega}{2\epsilon_0} P(\omega), \]

where \(P(\omega)\) is the slowly oscillating term of the induced polarization

\[ P(\omega) = 2N(\mu_{31}\rho_{13} + \mu_{32}\rho_{23}), \]

with \(N\) the atomic density of the medium. For a perfectly tuned cavity, the boundary conditions in the steady-state limit are [25]

\[ E(0) = \sqrt{T} E_I + RE(L), \]

\[ E_T = \sqrt{T} E(L), \]

where \(R\) and \(T\) are the reflectance and the transmittance of the semisilvered mirror (see figure 4), and \(E_I\) and \(E_T\) are the incident and the transmitted fields, respectively. Using the boundary conditions (24) and (25) we obtain the mean-field state equation

\[ \frac{\partial E(t)}{\partial t} = \kappa \left( \frac{E_I}{\sqrt{T}} - E(t) \right) + \frac{\omega}{\epsilon_0} N(\mu_{31}\rho_{13} + \mu_{32}\rho_{23}), \]

Figure 4. Unidirectional ring cavity with an atomic sample of length \(L\). \(E_I\) and \(E_T\) are the incident and transmitted fields, respectively. The total length of the cavity is \(L_T\).
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where $\kappa = c T / L$. In order to compare the results with the two-level atoms we define the cooperation parameter $C$ as usual [25]

$$C = \frac{\omega_{13}^2 \mu_{13}^2 N L}{\hbar \gamma_1 \epsilon_0 c T}.$$  \hspace{1cm} (27)

We calculate the steady state of the matter–radiation system, that is, we take the derivatives of equations (13)–(17) and (26) equal to zero to obtain the steady state of the transmitted field

$$y = x_1 - 2C \sqrt{\gamma_1 \gamma_2} \left[ \frac{\rho_{13} + \mu_{13} \rho_{23}}{\mu_{31}} \right].$$  \hspace{1cm} (28)

where we have defined the dimensionless incident field as $y = \mu_{31} E_1 / (\hbar \sqrt{T \gamma_1 \gamma_2})$. We consider again the case with $\gamma_1 = \gamma_2$ and $|\mu_{31}| = |\mu_{23}|$, so then $x_1 = x_2 = x$. Figure 5 gives the bistable behaviour of the three-level system with ($p = 1$) and without ($p = 0$) quantum interference. It can be seen that quantum interference leads to a significative decreasing of the bistability threshold ($\gamma_n$) by two orders of magnitude for the considered case, $C = 100$, $2 \epsilon_0 / \sqrt{\gamma_1 \gamma_2} = 0.05$ and $\delta_1 = 0.001$, i.e.

$$\gamma_n(p = 0) \approx 100, \quad \gamma_n(p = 1) = 500.$$  \hspace{1cm} (29)

To check the bistable behaviour of our system, we have solved numerically the coupled equations (13)–(17) and (26). It is found that only positive-slope regions of the transmission curves (output field $x$ versus incident field $y$) appear in the steady state. This is the well known criterion in most cases of optical bistability. In order to explain the different behaviour between the cases with and without quantum interference we shall analyse the nonlinear absorption response. As we mentioned in the previous section, the imaginary part of the optical coherences versus the detuning, $\delta_1$, shows a strong decreasing in a narrow region around the condition $\delta_1 = 0$ (or $\Delta_1 = - \Delta_2$) when the quantum interference is taken into account ($p = 1$) (see figure 2b). In this case, equation (28) gives $y = x$, that is, the system is transparent to the radiation and it is impossible for bistability to occur. However, the width of the dip increases as the field increases. Thus, if $\delta_1$ is slightly different to zero it can be seen that at low values of the driving field the atom absorbs and there is a low transmission. When the driving field increases, i.e. $x = 0.3$ in figure 2(b), the dip broadens and the atom is almost in a non-absorbing coherent superposition of the two upper sublevels. As a result of this, the atomic system becomes transparent. On the other hand, for $p = 0$ (see figure 2a) the dip and the non-absorbing superposition disappear. In this case, the transparency is allowed only by means of the usual saturation of the optical transitions. Thus the threshold field is much greater than the threshold found when quantum interference is present.

Another interesting feature of this atomic system is that optical bistability can be achieved with a cooperative parameter $C$ lower than the minimum required for the case of the two-level atoms where $C$ must satisfy $C > 4$. In order to compare the cases $p = 0$ and $p = 1$, figures 6(a)–(d) show a plot of the cavity field $x$ as a function of the driving field $y$ for $C = 1.5$ and different detunings. In all cases OB does not occur for $p = 0$ because $C < 4$. For $\delta_1 = 0$ no OB occurs in the system with $p = 1$ because the atom is always in a non-absorbing trapped state, but for $\delta_1 \neq 0$ (see figures 6b, c and d) a bistable behaviour appears with $p = 1$. The threshold depends on the value of the detuning.

In summary, if quantum interference is taken into account, $V$-type three-level atoms give a similar bistable behaviour to the $A$-type atoms [7, 8].

4. Linewidth effect on optical bistability

As we have just seen, in the case $p = 1$, the absorptive response of the $V$-type three-level atomic system is very sensitive to the detuning. It is well known that phase fluctuations may influence the optical properties of atomic systems [16]. Particularly, in $V$-type atoms it has been shown that phase fluctuations may destroy the trapped state. This suggests that phase fluctuation of the driving field can modify or eliminate the bistable response based on quantum interference. However, Gong et al [18] have shown that phase fluctuations
δk

Wiener characterized by the following random force [26]:

\[ \langle F(t)F(t) \rangle = 0, (30) \]

\[ \langle \phi(t)M_0 \cdot X + V, (34) \]

\[ F(t)F(t') = 2D\delta(t-t'). (34) \]

For the new variables defined in (30)–(32), the Bloch equations (5)–(9) become

\[ \frac{\partial \rho_{11}}{\partial t} = -\gamma_1 \rho_{11} - \gamma_2 (\rho_{12} + \rho_{21}) + i\Omega'_1 \rho_{13} - i\Omega'_2 \rho_{13} \]

\[ \frac{\partial \rho_{22}}{\partial t} = -\gamma_2 \rho_{22} - \gamma_1 (\rho_{12} + \rho_{21}) + i\Omega'_2 \rho_{32} - i\Omega'_2 \rho_{32} \]

\[ \rho_{13} = \rho'_{13} \exp(i\phi(t)), \]

\[ \rho_{23} = \rho'_{23} \exp(i\phi(t)), \]

\[ \Omega_j = \Omega'_j \exp(i\phi(t)), \]

\[ \Omega'_{13} = \gamma_2 \rho_{32} - i\Omega'_2 \rho_{32}, \]

\[ \Omega'_{23} = -\gamma_2 \rho_{13} + i\Omega'_2 \rho_{13}, \]

\[ \phi(t) \]

\[ \phi(t) \]

\[ (F(t)) = 0. \]
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where the steady-state values for $\langle \rho_{13} \rangle$ and $\langle \rho_{23} \rangle$ can be obtained from equation (42). In the rest of the work we shall consider the case with $\gamma_1 = \gamma_2$ [24] and $|\mu_{31}| = |\mu_{32}|$, so then $x_1 = x_2 = x$. Figure 7 shows the effect of the phase fluctuations in the bistable response. We consider the situation of optical resonance, $\delta_k = 0$. In the absence of phase fluctuations ($D_n = 0$), $\operatorname{Im}(\rho_{13} + \rho_{23}) = 0$ and equation (43) gives $y = x$. In this case the atoms are in a trapped state and optical bistability is impossible. However, for moderate non-zero values of $D \ll \gamma_1, \gamma_2$, the bandwidth of the stochastic field dephases the atomic coherences $\rho_{13}$ and $\rho_{23}$, leading to the destruction of the trapping [17]. The atom absorbs and optical bistability appears. Here the presence of the phase fluctuations is essential to break the trapping condition. If fluctuations increase $D \simeq \gamma_1, \gamma_2$, so that they are of the order of the linewidth of the atomic transitions, a change from the bistable response due to quantum interference to a bistable response based on saturation takes place (see figures 8(a) and (b)). For example, for $D_n = 2$, the system is equivalent to a $V$-type system with $p = 0$ as can be seen comparing figures 8(a) and (b).

This change is explained by analysing the behaviour of the $\operatorname{Im}(\rho_{13} + \rho_{23})$. Figure 9 shows the influence of the diffusion constant $D$ in the dip’s appearance. For a monochromatic field ($D = 0$), a pronounced dip is obtained, which reaches zero value at $\delta_k = 0$. When $D$ increases, the depth of the dip decreases and the dip broadens. At large values of $D$, this dip disappears. In this case, quantum interference between the two decay channels does not take place and the mechanism responsible for the nonlinear response of the system is the saturation of the optical transitions.

5. Conclusions

In this paper we have investigated the behaviour of a $V$-type three-level atomic system in a ring cavity taking into account the possibility of quantum interference between the two decay channels from the two upper sublevels to the ground level. Specifically, we analyse the optical response of a cavity filled with this type of atom and we find that optical bistability can be achieved via a mechanism which exploits the trapped state produced by the quantum interference of the two decay channels. This type of bistability can be obtained with a considerably lower threshold intensity and lower cooperative parameter than in two-level atoms. The system is very sensitive to the detuning between the driven optical field and the optical transitions. Moreover, this detuning controls the threshold and the width of the hysteresis loop.

We have also investigated the influence of the phase fluctuations in the absorptive properties of the $V$-type system.

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1 For example, in the Rb atom the lifetimes of the transitions are very similar.
We have found how the phase diffusion of the field can seriously prevent the dip observed in the susceptibility Im(\(\rho_{13} + \rho_{23}\)), in accordance with the results obtained by other authors in the spontaneous-emission spectrum of V-type atoms [5]. In the situation of optical resonance, \(\delta_k = 0\), in the absence of phase fluctuations (\(D = 0\)), the atoms are in a trapped state and optical bistability is impossible. However, for moderate non-zero values of phase fluctuations (\(D \ll \gamma_1, \gamma_2\)), the bandwidth of the stochastic field dephases the atomic coherences \(\rho_{13}\) and \(\rho_{23}\), leading to the destruction of the trapping [17]. The atom absorbs and optical bistability appears. Moreover if fluctuations increase \(D \simeq \gamma_1, \gamma_2\), so that they are of the order of the linewidth of the atomic transitions, a change from the bistable response due to quantum interference to a bistable response based on saturation occurs.

In summary, if quantum interferences are taken into account, V-type three-level atoms give a similar bistable behaviour to the A-type atoms, since the quantum interference overcomes the problem of the relaxation of the coherence of the upper sublevels via spontaneous emission considered in usual V-type atoms [7, 8].

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