Optical bistability in $V$-type atoms driven by a coherent field in a broadband squeezed vacuum

M.A. Antón, Oscar G. Calderón *, F. Carreño

Escuela Universitaria de Óptica, Universidad Complutense de Madrid, C/Arcos de Jalón s/n, 28037 Madrid, Spain

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Abstract

We study the optical bistability (OB) induced in a coherently-driven $V$-type three-level atom, when the cancellation of the spontaneous emission produced by quantum interference in the two possible decay channels to the ground sublevel takes place. In addition the atoms interact with a beam in a broadband squeezed vacuum state. New features are found in this situation. In particular, vacuum-induced optical bistability (VIOB) may be controlled by the amplitude of the squeezed vacuum or its phase relative to the phase of the external field.

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1. Introduction

In recent years, squeezed light sources have become available in laboratories and attention has turned to their interaction with optical systems. The interaction of atomic systems with squeezed light has been a subject of intense activity since Gardiner [1] showed that the two dipole quadratures of a two-level atom interacting with a squeezed vacuum decay at different rates. As a result, certain atomic properties such as the steady-state atomic polarization, saturation intensity and fluorescent spectrum of two-level atoms, change and become phase dependent. For a review about this issue see Refs. [2,3]. Over the past decade the study was mainly concentrated on two-level atoms and a large number of other effects have been predicted: subnatural spectral lines [4], anomalous spectral features (hole burning, and dispersive profiles at the line center) [5], amplification without inversion [6] and a linearly-dependent two-photon absorption rate [7–13]. The latter has been recently experimentally confirmed [14]. The effects of finite-bandwidth squeezed light on the fluorescence spectra have also been studied [15].
Moreover, during last years interest has been significantly focused on the quantum interference and coherence effects in multilevel atom systems induced by coherent electromagnetic fields [16]. A lot of phenomena such as electromagnetically induced transparency (EIT) [17–23], lasing without inversion (LWI) [24–27], refractive index enhancement without absorption [28–32], giant non-linearity [33–36], etc., have been predicted and experimentally demonstrated. The role of these atomic coherences in the context of collective phenomena, such as optical bistability (OB), has been subject to analysis. In earlier works, Walls et al. [37,38] proposed a novel scheme for optical bistability using atomic coherence effects in three-level systems. The OB in $\Lambda$-type atoms was found to be originated by population trapping in a coherent superposition of the ground state sublevels. This coherent superposition is not coupled to the excited level which leads to a strong narrow non-absorption resonance in the absorption profile. The non-absorption resonance dip broadens when the intensity of the driving field increases. The main feature of this mechanism is the following: it does not require atomic saturation and OB is achieved with low laser intensities. This interesting topic, the dynamics of $\Lambda$-type atoms in ring cavities, has been studied theoretically by Harshawardham et al. [39] and experimentally by Wang et al. [40].

On the other hand, in $V$-type atoms with large upper sublevel splitting, a similar dip in the absorption profile occurs, although in this case, the coherence between upper sublevels is reduced by spontaneous emission to the ground level and large intensities are necessary in order for quantum coherence to influence the absorption process. However, another way of generating coherence is connected with the relaxation processes such as spontaneous emission. It is well-known how quantum coherence can be created in interactions involving a common bath with a set of closely lying states. This rather contraintuitive phenomenon has been shown by Agarwal [41–48].

If the two upper levels are very close and damped by the usual vacuum interactions, spontaneous emission cancellation can take place, which offers the possibility to trap population in the excited levels when some particular conditions hold [49–54]. In this case the problem of the spontaneous emission can be avoided or minimized when considering quantum interference between upper sublevels. Quantum interference between the two transition channels connecting the ground-level is responsible for many novel effects, such as narrow resonances and probe transparency [50], phase dependent line shapes [53,54], dark spectral lines [55,56], etc. We have shown in a previous work that in this case a new kind of OB, vacuum-induced optical bistability (VIOB), in $V$-type atoms is achieved with a considerably lower threshold intensity, and a lower cooperative parameter than in two-level and $\Lambda$ three-level atoms [57]. The optical response was found to be very sensitive to the detuning between the driving field and the optical transitions. An interesting result pointed out in that work was that trapping condition, responsible of OB, is dramatically modified when considering phase fluctuations of the driving field. Therefore, one could expect that interaction of phase-dependent reservoirs, such as a squeezed vacuum one, with $V$-type atoms will also alter the trapping condition, and the OB consequently.

The effects of a squeezed vacuum input on OB have been studied by Galatola et al. [58] and Bergou and Zhao [59] in two-level atoms. Sing et al. [60] analyzed the influence of squeezed vacuum in intrinsic OB. Recently, Du et al. [61] have investigated the OB in $\Lambda$-type atoms. They showed that squeezed vacuum increases the range of the optical bistability when the squeezed photon number and the strength of two-photon correlation vary. Chen et al. [62,63] have studied OB in $V$-type atoms only in the degenerate case, and have shown that OB can be obtained with a cooperative parameter smaller than the usual value in two-level atoms.

In this Letter we analyze the dynamics of $V$-type atoms with closely spaced doublet interacting with an external coherent field and a squeezed vacuum field under conditions where vacuum-induced interference among decay channels are important. Specifically, we are interested in the influence of the squeezed vacuum on the non-absorption resonance, responsible for bistable response without saturation or VIOB. In the absence of squeezed...
vacuum, a sharp dip is obtained in the susceptibility profile. When the squeezed field is present, we find that its phase significantly modifies the width and the depth of the dip, switching the atomic system from transparency to absorption.

The Letter is organized as follows: Section 2 establishes the model, i.e., the Hamiltonian of the system and the evolution equations of the atomic operators assuming the rotating wave approximation. Section 3 deals with Maxwell’s equation in a ring cavity and the interplay between quantum interference and the squeezed field in the bistable response of the system. The influence of the squeezed vacuum on the coherences is studied in Section 4 where we show how the squeezed field partially destroys the trapping condition by means of the dressed state representation. The main conclusions are summarized in Section 5.

2. The model

We consider a $V$-type atom consisting of two upper sublevels $|3⟩$, $|2⟩$ coupled to a single ground level $|1⟩$ by a single-mode laser field. The energy-level scheme is shown in Fig. 1(a). The external coherent field $\hat{E}$ is taken linearly polarized in the same direction as the transition dipole moments and propagates in the $z$ direction. For this reason, we will omit the vectorial character of the different variables. The external field $\hat{E}$ is given by

$$\hat{E} = \frac{1}{2}E(z,t)e^{-i\omega_L t} + \text{c.c.},$$

(1)

$E(z,t)$ and $\omega_L$ being the slowly varying field envelope and the angular frequency of the field, respectively. Spontaneous and stimulated emissions between these states are governed by the interaction of the atom with a reservoir in a multimode squeezed state. In order to take into account the induced-coherence effects by spontaneous emission, the two upper levels $|3⟩$ and $|2⟩$ are coupled by the same vacuum modes to the ground level $|1⟩$. The resonant frequencies between the upper levels $|3⟩$, $|2⟩$ and the ground level $|1⟩$ are $\omega_{31}$ and $\omega_{21}$, respectively. Note that $\omega_{31} - \omega_{21} = \omega_{32}$, $\omega_{32}$ being the frequency separation of the excited levels.

The Hamiltonian of the system in the rotating wave approximation is given by [64]

$$H = \hbar \sum_{m=1}^{3} \omega_m |m⟩⟨m| + \hbar \sum_{k\lambda} \omega_{k\lambda} a_k^+ a_{k\lambda} + \hbar \sum_{m=2}^{3} \sum_{k\lambda} g_{mk} |m⟩⟨1| a_{k\lambda} + \text{h.c.} - \hbar \sum_{m=2}^{3} \Omega_m e^{-i\omega_m t} |m⟩⟨1| - \text{h.c.}$$

(2)

Fig. 1. (a) A $V$-type atom driven by a single-mode laser of frequency $\omega_L$. $\gamma_3$ and $\gamma_2$ are the decay rates from the excited sublevels to the ground level, and $\delta_k$ is the detuning of the coherent field with the central frequency (from the middle point of the two upper levels to level $|1⟩$). (b) Unidirectional ring cavity with an atomic sample of length $L$. $E_i$ and $E_T$ are the incident and transmitted fields, respectively. The total length of the cavity is $L_T$. $a$ represents the injected squeezed vacuum field with $\langle a \rangle = 0$. 
Here, $\hbar \omega_m$ are the energies of the atomic levels, $a_{k\lambda}$ ($a_{k\lambda}^\dagger$) is the annihilation (creation) operator of the $k$th mode of the vacuum field with polarization $\vec{e}_{k\lambda}$ ($\lambda = 1, 2$) and angular frequency $\omega_{k\lambda}$. The parameter $g_{mk}$ is the coupling constant of the atomic transition $|m\rangle \rightarrow |1\rangle$ with the electromagnetic mode

$$g_{mk} = -\frac{\omega_{k\lambda}}{\sqrt{2\hbar e_0 V}} (\mu_{1m} \cdot \vec{e}_{k\lambda}),$$

(3)

where $\mu_{1m}$ is the dipolar moment of the atomic transition $|m\rangle \rightarrow |1\rangle$ and $\Omega_m = \mu_{1m} E/(2\hbar)$ is the Rabi frequency of the transition $|m\rangle \rightarrow |1\rangle$.

We now assume that the quantized radiation field is in a broadband squeezed vacuum state with carrier frequency $\omega$ which is tuned close to the frequency of the atomic transitions $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$, that is, $2\omega \simeq \omega_{13} + \omega_{23}$. The bandwidth of the squeezing is assumed to be broad enough for the squeezed vacuum to appear as $\delta$-correlated squeezed white noise to the atom. The correlation function for the field operators $a(\omega_{k\lambda})$ and $a^+(\omega_{k\lambda})$ can be written as \[1,65,66\]

$$\langle a(\omega_{k\lambda})a^+(\omega_{k\lambda}')\rangle = \left[N(\omega_{k\lambda}) + 1\right] \delta(\omega_{k\lambda} - \omega_{k\lambda}'),$$

(4)

$$\langle a^+(\omega_{k\lambda})a(\omega_{k\lambda}')\rangle = N(\omega_{k\lambda})\delta(\omega_{k\lambda} - \omega_{k\lambda}'),$$

$$\langle a(\omega_{k\lambda})a(\omega_{k\lambda}')\rangle = M(\omega_{k\lambda})\delta(2\omega - \omega_{k\lambda} - \omega_{k\lambda}'),$$

(5)

where $N(\omega_{k\lambda})$ and $M(\omega_{k\lambda})$ are the slowly varying functions of the frequency and characterize the squeezing. The following inequality holds

$$|M(\omega_{k\lambda})|^2 \leq N(\omega_{k\lambda})N(2\omega - \omega_{k\lambda}) + \min\left[N(\omega_{k\lambda}), N(2\omega - \omega_{k\lambda})\right].$$

(6)

Note that $M$ is a complex magnitude so that $M(\omega_{k\lambda}) = |M(\omega_{k\lambda})|\ e^{i\phi_v}$, where $\phi_v$ is the phase of the squeezed vacuum. For $|M(\omega_{k\lambda})| = 0$, Eq. (4) describes a thermal field at temperature $T$, where $N(\omega_{k\lambda})$ is the mean occupation number of the mode $k\lambda$ with frequency $\omega_{k\lambda}$.

The system is studied using the density-matrix formalism. Following the traditional approach of Weisskopf and Wigner [64,67–70], we obtain the master equation for the reduced density matrix $\rho_I^t$ for the atomic system in the Born and Markov approximation. In the interaction picture the master equation is given by

$$\frac{\partial \rho_I^t}{\partial t} = -\frac{i}{\hbar}\left[H_{ex}^t, \rho_I^t\right]$$

$$-\frac{1}{2}\sum_{i,j=2}^3 (N(\omega_{i1}) + 1) \gamma_{ij} \left[(S_i^+ S_j^- \rho_I^t - S_j^- S_i^+ \rho_I^t) e^{i\omega_{ij}t} + (\rho_I^t S_j^+ S_i^- - S_i^- \rho_I^t S_j^+) e^{-i\omega_{ij}t}\right]$$

$$-\frac{1}{2}\sum_{i,j=2}^3 N(\omega_{i1}) \gamma_{ij} \left[(S_i^+ S_j^- \rho_I^t - S_j^- S_i^+ \rho_I^t) e^{i\omega_{ij}t} + (\rho_I^t S_j^+ S_i^- - S_i^- \rho_I^t S_j^+) e^{-i\omega_{ij}t}\right]$$

$$-\frac{1}{2}\sum_{i,j=2}^3 M(\omega_{i1}) \eta_{ij} \left[(S_i^+ \rho_I^t S_j^- - S_j^- \rho_I^t S_i^+) e^{i\omega_{ij}t} + (S_i^+ S_j^+ \rho_I^t - \rho_I^t S_j^+ S_j^+\right) e^{-i\omega_{ij}t}\right]$$

$$-\frac{1}{2}\sum_{i,j=2}^3 M^*(\omega_{i1}) \eta_{ij} \left[(S_i^+ \rho_I^t S_j^- - S_j^- \rho_I^t S_i^+) e^{i\omega_{ij}t} + (S_i^+ S_j^+ \rho_I^t - \rho_I^t S_j^+ S_j^+\right) e^{-i\omega_{ij}t}\right].$$

(7)

In the above equation we have introduced the usual shorter notation for the atomic operators, i.e.,

$$S_z^+ = (S_z^-)^\dagger = |2\rangle\langle 1|,$$

$$S_z^- = (S_z^+)^\dagger = |3\rangle\langle 1|.$$
and the coefficients $\gamma_{ij}$ and $\eta_{ij}$ are defined as [65,66]

$$
\gamma_{ij} = \pi g_i(\omega_1)g_j(\omega_1), \quad \eta_{ij} = \pi g_i(\omega_1)g_j(2\omega - \omega_1),
$$

where $\gamma_{ij} = \gamma_{ji}$ and $\eta_{ij} = \eta_{ji}$. The coefficients $\gamma_{ii} \equiv \gamma_i$ ($i = 2, 3$) in Eq. (6) are the decay rates for the $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$ transitions. Obviously in the radiative approximation the decay rate for $|3\rangle \rightarrow |2\rangle$ is $(\gamma_2 + \gamma_3)/2$.

The additional damping terms are $\gamma_{ij}$ ($i \neq j$). They are particularly important when $\omega_{32} \simeq \gamma_2$, $\gamma_3$ and they arise due to the coupling of transitions $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$ with the same vacuum mode, and are responsible for the quantum interference between the two channels [71–74]. It can be seen that this term oscillates at the frequency difference $\Delta = \omega_{13} - \omega_{11}$, so that if $\Delta$ is large, they may drop. This is the case treated in Ref. [72]. The present discussion is based on the situation in which $\omega_{13} \simeq \omega_{11}$, so such non-secular terms must be retained. Moreover, the presence of squeezing and the fact that $\langle a a \rangle \neq 0$ introduce the additional damping constants $\eta_{ij}$ which oscillate at $2\omega_3 - \omega_{13} - \omega_{11}$. It must be noted that these terms disappear ($\eta_{22} = \eta_{33} = 0$) in a ladder configuration [14], and in a V-type atomic configuration when $\omega_{31} \gg \omega_{21}$ [72]. In the last case, the atomic operators do not depend on correlations between pairs of modes, and this leads to absence of phase sensitivity in population decay. However, in the V-type atomic configuration considered here, the central frequency of the squeezed vacuum is near the center of the doublet so $2\omega_3 \simeq \omega_{13} + \omega_{11}$ and all terms $\eta_{ij}$ have to be retained. The main consequence of this fact is that some optical properties of a V atom with closed sublevels become phase dependent as we will show later.

Finally, $H_{ex}^I$ represents the interaction between the atom and the external driving field in the interaction picture

$$
H_{ex}^I = \hbar \sum_{m=2}^{3} \Omega_m e^{-i(\omega_m - \omega_0) t} |m\rangle \langle 1| + h.c.
$$

The radiative shifts (Lamb and Stark shifts) have been ignored. In addition, it can be shown [41–47] that

$$
\gamma_{23} = \eta_{23} = \sqrt{\gamma_{22}} \gamma_{33} \left( \frac{\mu_{13} \cdot \hat{\mu}_{12}}{\mu_{13} \mu_{12}} \right),
$$

where the transition dipole moments $\mu_{13}$ and $\mu_{12}$ are assumed to be real valued. The quantum interference is maximum if the transition moment $\mu_{13}$ is parallel to $\mu_{12}$, but it disappears if they are orthogonal. Several methods to bypass this stringent condition have been proposed. For example, the two level may be mixed by applying static and electromagnetic fields so that the relevant dipole moments become nonorthogonal [75]. In a very recent paper [76], Agarwal demonstrated that the anisotropy of the vacuum of the electromagnetic field could lead to quantum interference among the decay channels of closely lying states even in the dipole matrix elements were orthogonal. These methods are illustrated in a recent review [16]. Now we eliminate the explicit temporal dependence of the density matrix equation through an appropriate unitary transformation $\rho = U_N^{-1} \rho_U U_N$, where

$$
U_N = \exp\left[i(\Delta_3 |3\rangle \langle 3| + \Delta_2 |2\rangle \langle 2|) t\right],
$$

$\Delta_m = (\omega_m - \omega_L) (m = 2, 3)$ being the laser detuning from the resonance with the state $|m\rangle$. In this frame, the density matrix equation of the system can be written as

$$
\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H_{ex}, \rho] - \frac{i}{\hbar}[H_0, \rho]
\left[ -\frac{1}{2} \sum_{i,j=2}^{3} \left[ N(\omega_{11}) + 1 \right] \gamma_{ij} \left[ (S_i^+ S_j^- \rho - S_j^- \rho S_i^+) + (\rho S_i^+ S_j^- - S_i^- \rho S_j^+) \right] \right]
\left[ -\frac{1}{2} \sum_{i,j=2}^{3} N(\omega_{11}) \gamma_{ij} \left[ (S_i^- S_j^+ \rho - S_j^+ \rho S_i^-) + (\rho S_i^- S_j^+ - S_i^+ \rho S_j^-) \right] \right]

\text{with}
$$

\[\rho = \begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{13} \\
\rho_{21} & \rho_{22} & \rho_{23} \\
\rho_{31} & \rho_{32} & \rho_{33}
\end{pmatrix}
\]
In addition, all frequencies and detunings of the problem will be normalized to \( \Omega_m = |\Omega_m| e^{-i\Phi_L} \), and the Hamiltonians \( H_{\text{ex}} \) and \( H_0 \) are

\[
H_{\text{ex}} = -\hbar \sum_{i,j=1}^{3} |\Omega_m|^2 |i\rangle \langle i| - \text{h.c.,} \quad H_0 = \hbar \Delta_3 |3\rangle \langle 3| + \hbar \Delta_2 |2\rangle \langle 2|.
\]

We assume that \( \omega \approx \omega_L \), \( N(\omega_{21}) = N(\omega_{31}) \equiv N \), and \( M(\omega_{21}) = M(\omega_{31}) \equiv M \). Then, the evolution equations of the density matrix elements in the rotating frame take the form

\[
\frac{\partial \rho_{33}}{\partial \tau} = -\left(N + \frac{1}{2}\right) \rho_{33} - (N + 1) p(\rho_{23} + \rho_{32}) + \frac{2N}{\alpha} \rho_{11} + i|\alpha| x_3 \rho_{13} - i|x_3| \rho_{31},
\]

\[
\frac{\partial \rho_{22}}{\partial \tau} = -2(N + 1) \alpha \rho_{22} - (N + 1) p(\rho_{32} + \rho_{23}) + 2N \alpha \rho_{11} + i|x_2| \rho_{12} - i|x_2| \rho_{21},
\]

\[
\frac{\partial \rho_{31}}{\partial \tau} = -F_{31} \rho_{31} - (N + 1) p \rho_{21} - 2|M| e^{i\Phi} \left( \frac{1}{\alpha} \rho_{13} + p \rho_{12} \right) - i|x_3| (2\rho_{33} + \rho_{22} - 1) - i|x_2| \rho_{32},
\]

\[
\frac{\partial \rho_{21}}{\partial \tau} = -F_{21} \rho_{21} - (N + 1) p \rho_{31} - 2|M| e^{i\Phi} (\alpha \rho_{12} + p \rho_{13}) - i|x_2| (\rho_{33} + 2\rho_{22} - 1) - i|x_3| \rho_{23},
\]

\[
\frac{\partial \rho_{32}}{\partial \tau} = -F_{32} \rho_{32} - (N + 1) p(\rho_{23} + \rho_{31}) + 2N p \rho_{11} + i|x_2| \rho_{13} - i|x_3| \rho_{21},
\]

where we have introduced a dimensionless time \( \tau \), and dimensionless variables

\[
\tau = \frac{\sqrt{2} \gamma_3}{\gamma_2} t,
\]

\[
\alpha = \frac{\sqrt{2}}{\sqrt{\gamma_3}},
\]

\[
x_j = \frac{2}{\sqrt{\gamma_3} \gamma_2} \Omega_j,
\]

\[
p = \frac{1}{\sqrt{\gamma_3} \gamma_2} \gamma_2.
\]

In addition, all frequencies and detunings of the problem will be normalized to \( \sqrt{\gamma_3} / 2 \), and we have introduced the following normalized detunings:

\[
F_{31} = \left[(N + 1) \frac{1}{\alpha} + N \left( \alpha + \frac{1}{\alpha} \right) + i \frac{2}{\sqrt{\gamma_3} \gamma_2} \Delta_3 \right],
\]

\[
F_{21} = \left[(N + 1) \alpha + N \left( \alpha + \frac{1}{\alpha} \right) + i \frac{2}{\sqrt{\gamma_3} \gamma_2} \Delta_2 \right],
\]

\[
F_{23} = \left[(N + 1) \left( \alpha + \frac{1}{\alpha} \right) + i \frac{2}{\sqrt{\gamma_3} \gamma_2} (\Delta_2 - \Delta_3) \right],
\]

where \( F_{ij} = F_{ji}^* \). The phase \( \Phi = 2\phi_L - \phi_e \) is the phase difference between the coherent field and the squeezed field. An inspection of Eq. (14) reveals several terms which are not present in the usual density-matrix equations of \( V \)-type three-level systems in ordinary vacuum [37]. The squeezed vacuum introduces a phase-dependent relaxation
process in the polarization proportional to the $|M| e^{i\Phi}$ in the equations of $\rho_{31}$ and $\rho_{21}$. This fact appears clearly if we analyze the decays of the two quadratures, $S_x \equiv \rho_{13} + \rho_{31}$ and $S_y \equiv i(\rho_{13} - \rho_{31})$ of the atomic polarization. When $\gamma_3 = \gamma_2$ and $\Delta_3 = \Delta_2 = 0$, it is easy to obtain the equations of motion of quadratures from Eq. (14) as

$$\frac{\partial S_x}{\partial \tau} = \left[(3N + 1) + (N + 1)p + 2|M|(1 + p)\cos \Phi\right] S_x - 2|M|(1 + p)\sin \Phi S_y + \text{remaining terms},$$

$$\frac{\partial S_y}{\partial \tau} = -2|M|(1 + p)\sin \Phi S_x - \left[(3N + 1) + (N + 1)p - 2|M|(1 + p)\cos \Phi\right] S_y + \text{remaining terms}. \quad (22)$$

that is, quadratures decay at the rates $\gamma_x$ and $\gamma_y$ given by

$$\gamma_x = (3N + 1) + (N + 1)p + 2|M|(1 + p)\cos \Phi, \quad (23)$$

$$\gamma_y = (3N + 1) + (N + 1)p - 2|M|(1 + p)\cos \Phi. \quad (24)$$

The atomic polarization decays at a different rate which depends on $N$, $|M|$ and $\Phi$, and therefore, the atomic coherences dephase each other. Moreover, when the quantum interference is maximal ($p = 1$), the difference between both decay rates is maximal too, i.e., quantum interference enhances the effect of the squeezed field.

In the rest of this Letter, we will consider a perfect squeezing condition, i.e., $|M| = \sqrt{N(N + 1)}$.

3. Effect of the squeezed field on the bistable response

In order to study the role of the squeezed vacuum on the bistable response based on quantum interference (VIOB) in $V$-type three-level atoms, let us consider a medium of length $L$ composed by this type of atoms embedded in a unidirectional ring cavity with total length $L_T$ (see Fig. 1(b)). Our system is similar to the one discussed by Galatola et al. [58] and Bergou et al. [59] in the context of two-level atoms. The incident coherent field $E_I$ is taken linearly polarized in the same direction as the transition dipole moments and propagates in the $z$ direction. For this reason, we will omit the vectorial character of the different variables. The field $E$ is given by Eq. (1). The propagation of the laser field in the medium is governed by Maxwell’s wave equation, which in the slowly varying envelope approximation reduces to

$$c \frac{\partial E(z,t)}{\partial z} + \frac{\partial E(z,t)}{\partial t} = \frac{i \omega_l}{2\epsilon_0} P(\omega_L), \quad (25)$$

where $P(\omega_L)$ is the slowly oscillating term of the induced polarization

$$P(\omega_L) = 2N_a(\mu_{13}\rho_{31} + \mu_{12}\rho_{21}). \quad (26)$$

$N_a$ being the atomic density of the medium. For a perfectly tuned cavity, the boundary conditions in the steady-state limit are [59, 77]:

$$E(0) = \sqrt{T} E_I + R E(L), \quad (27)$$

$$E_T = \sqrt{T} E(L), \quad (28)$$

where $R$ and $T$ are the reflectance and the transmittance of the semisilvered mirror (see Fig. 1(b)), and $E_I$ and $E_T$ are the incident and the transmitted fields, respectively. Using the boundary conditions (27), (28) we obtain the mean-field state equation

$$c \frac{\partial E(t)}{\partial t} = \kappa \left( \frac{E_I}{\sqrt{T}} - E(t) \right) + i \frac{\omega_l}{\epsilon_0} N_a(\mu_{13}\rho_{31} + \mu_{12}\rho_{21}), \quad (29)$$

where $\kappa = cT/L_T$. Now, we define the cooperation parameter $C$ as usual [77]

$$C \equiv \frac{\alpha_{31}\mu_{13}^2 N_a L}{\hbar \gamma_3\epsilon_0 c T}. \quad (30)$$
Fig. 2. Transmitted normalized field $|\xi_3|$ vs. the incident one $|\eta|$ for $\delta_k = 0$ (solid line), $\delta_k = 1 \times 10^{-4}$ (dotted line), and $\delta_k = 1 \times 10^{-3}$ (dashed line). The rest of parameters are $C = 100$, $2\omega_3\sqrt{\gamma_2/\gamma_3} = 0.03$, and $\gamma_2 = \gamma_3$. Case without squeezed vacuum, $N = 0$. Comparison between (a) $p=1$ and (b) $p=0$.

Let us focus on the steady-state regime of the matter-radiation system, that is, we set the derivatives of Eqs. (14) and (29) equal to zero, and finally the steady-state of the transmitted field is given by

$$y = x_3 = 2iC \frac{\sqrt{\gamma_3}}{\gamma_2} \left[ \rho_{31} + \frac{\mu_{12}}{\mu_{13}} \rho_{21} \right],$$

(31)

where we have defined the dimensionless incident field as $y \equiv \mu_{13}E_I/\left(\hbar \sqrt{\gamma_3/\gamma_2} \right)$. In the following, we will analyze the dynamics as a function of the dimensionless detuning $\delta_k \equiv (\Delta_3 + \Delta_2)/\sqrt{\gamma_3/\gamma_2}$, which is the laser detuning from the resonance with the center of the two excited levels (see Fig. 1(a)). In the rest of the Letter, we will consider a dimensionless frequency separation of the excited levels $2\omega_3/\sqrt{\gamma_3/\gamma_2} = 0.03$. While the bistable response in V-type atoms has been considered in a few papers [61,62], the discussions were limited to a regime of input field where saturation took place. In contrast, in this Letter we focus on the OB due to quantum interference (VIOB). First of all, and for the sake of completeness, we briefly present the main features of the VIOB when the atoms are damped by the standard vacuum field, which has been treated by us in a previous work [57]. Fig. 2 presents the bistable curves with $(p = 1)$ and without $(p = 0)$ quantum interference. Note that quantum interference leads to a significative decrease of the bistability threshold $(y_{th})$ by two orders of magnitude for the considered case: $C = 100$, $\gamma_3 = \gamma_2$, and arbitrarily small value of the dimensionless detuning $\delta_k$. In the case with maximal quantum interference shown in Fig. 2(a) and by considering $\delta_k = 0$ (solid line), OB does not take place since the atom is in a non-absorbing trapped state. This behavior is based on the cancellation of the spontaneous emission produced by quantum interference in the two possible decay channels to the ground sublevel. When the driving field is detuned from the average atomic frequency $(\delta_k \neq 0)$, a bistable response appears (dotted and dashed lines in Fig. 2(a)). This type of bistable response was termed vacuum-induced optical bistability (VIOB) in our previous work [57]. In the absence of quantum interference $(p = 0)$, transparency is attained by means of the usual saturation of optical transitions (see Fig. 2(b)). Thus, the threshold field when $p = 0$ is much greater than the one found when quantum interference is present.

A trapped state can also be attained in the case of considering unequal spontaneous decay rates of the excited doublet $(\gamma_2 \neq \gamma_3)$. It has been shown in Refs. [41–47] that in order to produce a trapped state, the driving field has
Fig. 3. Transmitted normalized field $|x_3|$ vs. the incident one $|y|$ for $\delta_k = 0$ (solid line), and $\delta_k = \delta_k^{\text{trap}} = 0.0123$ (dotted line) (see Eq. (32)). The rest of parameters are $C = 100$, $2\omega_{32} \sqrt{\gamma_2 \gamma_3} = 0.03$, $\gamma_2/\gamma_3 = 0.1$, and $p = 1$. Case without squeezed vacuum, $N = 0$.

to be tuned in such a way that the following relation holds,

$$\delta_k^{\text{trap}} = \frac{\omega_{32}}{\sqrt{\gamma_2 \gamma_3}} \left( \frac{2}{1 + \alpha^2} - 1 \right). \quad (32)$$

Thus, if $\delta_k \neq \delta_k^{\text{trap}}$, OB occurs as it is shown in Fig. 3 (solid line), whereas if $\delta_k = \delta_k^{\text{trap}}$, the atomic system becomes transparent (dotted line).

Now let us consider the effects of the squeezed vacuum on the VIOB. The squeezed vacuum field introduces two important modifications in the evolution equations of the atom (14): it introduces terms due to the non-zero mean photon number—the terms proportional to $N$—and it couples $\rho_{31}$ with $\rho_{13}$ and $\rho_{12}$, through the squeezing parameter $M$. Thus, a phase dependence on the atomic relaxation processes appears. Fig. 4 shows the effect of the squeezed vacuum on the VIOB when quantum interference is maximal for two values of the ratio of the spontaneous decay rates. In both cases, the detuning was chosen as that leading to the trapping state, $\delta_k = \delta_k^{\text{trap}}$. In the absence of the squeezed field ($N = 0$), the non-absorption takes place, and Eq. (31) gives $y = x_3$. In this case the atoms are in a trapping state and OB does not take place. However, for non-zero values of $N$, the bandwidth of the squeezed vacuum leads to the partial destruction of the trapping. The atom absorbs and optical bistability appears (see Fig. 4). This relevant case was not considered in Ref. [63]. In other words, the presence of the squeezed vacuum is essential to break the trapping condition. Even a thermal field at finite temperature leads to the appearance of OB (not shown in Fig. 4). It is to be noted from Fig. 4, that an extremely weak squeezed field leads to VIOB. Effects induced by very low squeezed fields have been found in other problems. For example, Zhou et al. [78] found anomalous fluorescence spectra in two-level atoms interacting with an extremely weak squeezed vacuum. By looking at Fig. 4, we, clearly, observe that bistability threshold increases as the parameter $N$ becomes large. Furthermore, at very large values of $N$, a change from the bistable response due to quantum interference (VIOB), to a saturation-based bistable response occurs (not shown in Fig. 4).

As we have seen in Fig. 4, optical bistability based on quantum interference can be achieved independently of the ratio between the spontaneous decay rates when the external field is tuned close to $\delta_k^{\text{trap}}$. In the extreme case of a very low value of $\gamma_2/\gamma_3$ the detuning needed to produce the trapped state is approximately $\delta_k^{\text{trap}} \simeq \omega_{32}/\sqrt{\gamma_2 \gamma_3}$, i.e., $\Delta_3 \simeq \omega_{32}$ and $\Delta_2 \simeq 0$. Thus, the external field must be tuned nearly resonant with level 2 and the direct transition to this level is strongly diminished due to the small spontaneous decay rate $\gamma_2$. The effects of quantum interference, even in this case, become apparent on the atomic response, as it is shown in Fig. 5(a) for a ratio $\gamma_2/\gamma_3 = 0.01$. In
Fig. 4. Transmitted normalized field $|x_3|$ vs. the incident one $|y|$ for a squeezed parameter $N = 0$ (dotted line), $N = 5 \times 10^{-5}$ (solid line), $N = 5 \times 10^{-4}$ (dashed line), and $N = 1 \times 10^{-3}$ (dashed dotted line). The rest of parameters are $C = 100$, $2\omega_3 \sqrt{\gamma_3 \gamma_2} = 0.03$, and $\Phi = 0$. The detuning value corresponds to the trapping condition ($\delta_k^{\text{trap}}$) (a) $\delta_k = 0$ for $\gamma_2 = \gamma_3$ and (b) $\delta_k = 0.012$ for $\gamma_2/\gamma_3 = 0.1$.

Fig. 5. Transmitted normalized field $|x_3|$ vs. the incident one $|y|$ for a squeezed parameter $N = 0$ (dotted line) and $N = 1 \times 10^{-3}$ (solid line). The rest of parameters are $C = 5$, $2\omega_3 \sqrt{\gamma_3 \gamma_2} = 0.03$, $\gamma_2/\gamma_3 = 0.01$, and $\Phi = 0$. The detuning value corresponds to the trapping condition, $\delta_k^{\text{trap}} \simeq 0.015$. Comparison between (a) $p = 1$ and (b) $p = 0$. 
Fig. 6. (a) Transmitted normalized field $|x_3|$ vs. the incident one $|y|$ for a mean photon number $N = 0.1$ and phase difference $\Phi = 0$ (dotted line), $\Phi = \pi/2$ (dashed line), and $\Phi = \pi$ (solid line). (b) Transmitted normalized field $|x_3|$ as a function of the phase difference $\Phi$ at $N = 0.1$. The input field is $|y| = 3.25$. The parameters in (a) and (b) are: $C = 10$, $2\omega_{32}\sqrt{\gamma_2\gamma_3} = 0.03$, $\gamma_2 = \gamma_3$, $p = 1$, and $\delta_k = 0$.

Contrast, in the absence of quantum interference, OB does not take place even by considering high values of the input field (see Fig. 5(b)) since the laser field is far from resonance with the relevant upper level 3.

The influence of the squeezed phase $\Phi$ on VIOB is negligible when the amplitude of two-photon correlation $\langle |M| \rangle$ takes extremely low values as those used to produce Figs. 4 and 5. A simple inspection of Eqs. (23) and (24) reveals that $\gamma_x$ will differ enough from $\gamma_y$ when the term $2|M|(1 + p)\cos \Phi$ become noticeable. By increasing the value of $N$ in two orders of magnitude with regard to those previously used, which now is experimentally achievable in laboratories, and by choosing a cooperative parameter $C = 10$, we obtain the bistable curves shown in Fig. 6(a). The bistability threshold intensity diminishes as the phase $\Phi$ increases, and simultaneously the bistable region narrows and, eventually, disappears when $\Phi = \pi$. This behavior opens the possibility of inducing switching of the optical bistable system by varying the squeezed phase. In other words, by keeping constant the input field, it is possible to make the system to jump from a low to a high transmission state, and vice versa. This behavior is shown in Fig. 6(b). There we plot the behavior of the amplitude of the transmitted field when we adiabatically change the phase $\Phi$ of the squeezed control beam. The jumps are produced by the shift of the bistable curve as $\Phi$ changes, as shown in Fig. 6(a). This phenomenon resembles the results found in two-level atoms by Galatola et al. [58].

4. Effect of the squeezed field on the atomic system. Dressed state analysis

In order to shine a light on the mechanisms that lead to the results found in the previous section, we proceed in two ways: first at all we solve numerically the Eq. (14) with the aim of computing the absorption spectrum, and we analyze how the squeezed field influences it. Later we show that a convenient set of dressed atomic states can be used to obtain an asymptotic approximation which makes the physical processes transparent.

The steady-state behavior of the system is obtained by setting $\partial \rho_{nm}/\partial \tau = 0$, so that Eq. (14) reduces to a set of algebraic equations. The absorption characteristics of the optical coherences on the transition $|3\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |1\rangle$, i.e., $\text{Im}(\rho_{31} + \rho_{21})$, as a function of the detuning $\delta_k$ can be easily obtained. In the rest of the section we
will consider the case with equal spontaneous decay rates $\gamma_3 = \gamma_2$ [79], thus $\mu_{13} = \mu_{12}$ and $|x_3| = |x_2|$, for the case with maximal quantum interference, $p = 1$.

The spectrum is plotted in Fig. 7(a) for different squeezed field parameters. In the absence of the squeezed field ($N = 0$), a sharp dip appears with null absorption at zero detuning (dotted line). As $N$ increases the dip remains with a non-null absorption at $\delta_k = 0$, and it broadens. At very large values of $N$ the absorption dip disappears. In this situation, there is not a complete destructive interference between the two decay channels and the absorption behavior resembles to the case without quantum interference ($p = 0$). This fact is responsible of the appearance of VIOB at zero detuning when the squeezed is turned on.

In Fig. 7(b) we consider the special circumstance where $\delta_k = 0$. There we plot the absorption as a function of the mean photon number $N$. This figure clearly illustrates the destruction of the non-absorbing resonance. This behavior is easily explained in terms of the dressed states (see below).

The phase $\Phi$ also changes the shape and the depth of the dip, although its effect is less pronounced than that arising from changing the value of $N$. The spectrum is plotted in Fig. 8(a) for different values of $\Phi$ and keeping constant the value of $N$. It is to be noted that the absorption curve becomes asymmetric and its minimum slightly shifts to positive detuning values when $\Phi = \pi/2$. The absorption curve recovers its symmetric shape when $\Phi = \pi$.

In Fig. 8(b) the imaginary part of the polarization $\text{Im}(\rho_{31} + \rho_{21})$ vs. the phase $\Phi$ is shown at resonance. The curve exhibits an oscillatory behavior of the absorption. The origin of this behavior arises from the fact that the squeezed phase modulates the degree of quantum interference between the two decay channels. This modulation is responsible for the result depicted in Fig. 6(b).

In order to explain the main features of the numerical results we derive explicit expressions for the populations in the dressed state representation in the limit of large effective Rabi frequencies.

The dressed states are obtained by finding the eigenvectors of the interaction Hamiltonian, Eq. (13). We are interested in the case $\Delta_3 = -\Delta_2 = \omega_{32}/2$, that is, $\delta_k = 0$, and the magnitude of both dipole moments to be identical (so that $\gamma_2 = \gamma_3 = \gamma$ and $\Omega_2 = \Omega_3 = \Omega$). This case is relevant because the atomic system with quantum interference is in a trapped state and, consequently, bistability is impossible. In this situation, the eigenvalues and

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2 For example, in Rb atom the life-time of the transitions are very similar, see Ref. [79].
Fig. 8. (a) Imaginary part of the susceptibility \( \rho_{31} + \rho_{21} \) vs. the detuning \( \delta_k \) for \( N = 0.01 \) and a phase difference \( \Phi = 0 \) (dotted line), \( \Phi = \pi/2 \) (dashed line), and \( \Phi = \pi \) (solid line). (b) Imaginary part of the susceptibility \( \rho_{31} + \rho_{21} \) vs. \( \Phi \) at resonance (\( \delta_k = 0 \)) for \( N = 0.01 \). The rest of the parameters in (a) and (b) are as in Fig. 7.

eigenstates of the interaction Hamiltonian (13) are given by [80]

\[
\begin{align*}
|+\rangle &= \frac{1 + \epsilon}{2} |3\rangle + \frac{1 - \epsilon}{2} |2\rangle + 2\xi |1\rangle, \\
|b\rangle &= -2\xi |3\rangle + 2\xi |2\rangle + \epsilon |1\rangle,
\end{align*}
\]

with the corresponding energies \( \lambda_+ = \hbar \Omega R / 2 \), \( \lambda_b = 0 \) and \( \lambda_- = -\hbar \Omega R / 2 \), respectively, and where

\[
\Omega_R = \sqrt{\frac{\omega_{32}^2 + 8\Omega^2}{\Omega}}, \quad \xi = \frac{\Omega}{\Omega_R}, \quad \epsilon = \frac{\omega_{32}}{\Omega_R}.
\]

The projection of the master equation onto the dressed-state basis gives rise to complicated couplings between dressed-state populations and coherences. However, the situation can be simplified in the high-field limit where the effective Rabi frequency is much greater than all relaxation rates, i.e., \( \Omega_R \gg \gamma \). In this case, we can ignore the non-secular terms, i.e., coupling between populations and coherences because matrix elements associated with various frequencies may be omitted to order \( O(\gamma/\Omega_R) \). A lengthy but straightforward calculation permit us to obtain the Bloch equations in the dressed state representation as

\[
\begin{align*}
\frac{\partial \rho_{bb}}{\partial t} &= -\Gamma_1 \rho_{bb} + \Gamma_b, \\
\frac{\partial (\rho_{++} - \rho_{--})}{\partial t} &= -\Gamma_2 (\rho_{++} - \rho_{--}), \\
\frac{\partial \rho_{+b}}{\partial t} &= -\left( \Gamma_3 + i \frac{\Omega R}{2} \right) \rho_{+b} + \Gamma_4 \rho_{-b}, \\
\frac{\partial \rho_{b-}}{\partial t} &= -\left( \Gamma_3 + i \frac{\Omega R}{2} \right) \rho_{b-} + \Gamma_4^* \rho_{+b}, \\
\frac{\partial \rho_{+-}}{\partial t} &= -(\Gamma_5 + i \Omega R) \rho_{+-},
\end{align*}
\]
with

\[
\Gamma_0 = \frac{N + 1}{2} \left[ (\gamma + \gamma_{23}) \epsilon^2 + (\gamma - \gamma_{23}) \epsilon^4 \right] + \frac{N}{2} (\gamma - \gamma_{23})(1 - \epsilon^2) + |M|(\gamma - \gamma_{23})(1 - \epsilon^2) \epsilon^2 \cos \Phi,
\]

\[
\Gamma_1 = \frac{N + 1}{2} \left[ (\gamma + \gamma_{23}) \epsilon^2 + (\gamma - \gamma_{23})(3 \epsilon^4 - 4 \epsilon^2 + 2) \right] + \frac{N}{2} (\gamma - \gamma_{23})(3 \epsilon^4 - 4 \epsilon^2 + 1) + 4 \gamma \epsilon^2
\]

\[
+ 3|M|(\gamma - \gamma_{23})(1 - \epsilon^2) \epsilon^2 \cos \Phi,
\]

\[
\Gamma_2 = \frac{N + 1}{2} (\gamma + \gamma_{23} + (\gamma - \gamma_{23}) \epsilon^2) + N(1 - \epsilon^2) \gamma + (1 - \epsilon^2) |M|(\gamma + \gamma_{23}) \cos \Phi,
\]

\[
\Gamma_3 = \left\{ \begin{array}{l}
\frac{1}{2} \left[ N \left( 1 + \frac{3}{2} \epsilon^2 - 2 \epsilon^4 \right) + 1 + \frac{1}{2} \epsilon^2 - \epsilon^4 - 2(1 - \epsilon^2) \epsilon^2 |M| \cos \Phi \right] (\gamma - \gamma_{23}) \\
+ \frac{N + 1}{4} (\gamma + 2 \gamma_{23}) + \frac{N}{2} (\epsilon^2 + 1) \gamma,
\end{array} \right.
\]

\[
\Gamma_4 = -\frac{1}{2} (1 - \epsilon^2) \epsilon^2 (2N + 1) + M^* (1 - \epsilon^2) \epsilon^2 + M \epsilon^4 (\gamma - \gamma_{23}) + \frac{M}{2} \epsilon^2 (\gamma + \gamma_{23}),
\]

\[
\Gamma_5 = \frac{N + 1}{4} \left[ 3(\gamma + \gamma_{23}) + (\gamma - \gamma_{23}) \epsilon^4 - 4 \gamma_{23} \epsilon^4 \right] + (1 - \epsilon^4) \frac{N}{4} \epsilon^2 (\gamma_{23} - \gamma) + 5 \gamma + \gamma_{23}
\]

\[
- (1 - \epsilon^2) \frac{M}{2} \left[ (\gamma + \gamma_{23}) + (\gamma_{23} - \gamma) \epsilon^2 \right] \cos \Phi.
\]  

Equation (40) reduce to those obtained by Zhou and Swain in the case \( N = M = 0 \) [80]. In the steady-state, only the diagonal elements are non-zero:

\[
\rho_{++} = \rho_{--} = \frac{\Gamma_1 - \Gamma_0}{2 \Gamma_1}, \quad \rho_{bb} = \frac{\Gamma_0}{\Gamma_1}.
\]  

(41)

If squeezed vacuum is absent, \( N = M = 0 \), and quantum interference is maximum, i.e., \( \gamma_{23} = \gamma \) \((p = 1)\), the steady-state populations in Eq. (41) are \( \rho_{++} = \rho_{--} = 0 \), and \( \rho_{bb} = 1 \). In other words, the atom is trapped in the dressed state \( |b\rangle \). Nevertheless, when the atom is driven by a thermal or in a squeezed field, that is, \( N \neq 0 \), the steady-state populations in the dressed states are

\[
\rho_{bb} = \frac{N + 1}{3N + 1},
\]

(42)

\[
\rho_{++} = \rho_{--} = \frac{N}{3N + 1},
\]

(43)

that is, no population is trapped in a special dressed state, so no trapping occurs if the atom interacts with a squeezed bath. However, this analysis reveals that if quantum interference is not maximal, \( 0 < \gamma_{23} < \gamma \) \((0 < p < 1)\), the population in the dressed states depend on both \( N \) and the squeezing phase \( \Phi \). This explain qualitatively the dependence of the bistable response of the squeezing phase \( \Phi \) previously found.

5. Conclusions

This study completes and extends earlier investigations [57,61] on the response of \( V \)-type three-level atoms to external laser fields. It has been shown that this kind of atoms display a broad range of effects as a result of the interference effects between the two decay pathways from the excited doublet to the ground state that, under special circumstances, leads to the formation of a trapped state. The effect of a broadband squeezed vacuum has been shown to produce a partial destruction of the trapping state.

We have analyzed the optical response of a cavity filled with this type of atoms and we found that OB can be achieved via a mechanism which exploits the trapped state produced by the quantum interference of the two decay
channels, the so-called vacuum-induced optical bistability (VIOB). We can obtain absorption or transparency on optical transitions by tuning the mean number of photons of the vacuum mode, the relative phase of the coherent and vacuum field, or both.

Furthermore, we have studied the influence of the squeezed vacuum on the steady-state atomic behavior. We have shown how the squeezed vacuum field can modify the dip observed in the susceptibility \( \text{Im}(\rho_{13} + \rho_{12}) \). At optical resonance \((\delta_k = 0)\), and in the absence of the squeezed vacuum field, the atoms are in a trapped state, and optical bistability is impossible to occur. However, when the vacuum field is present, its bandwidth dephases the atomic coherences \( \rho_{31} \) and \( \rho_{21} \) leading to a partial destruction of the trapping. The atom absorbs and optical bistability appears. By using the atomic dressed state representation, we have explained the partial destruction of the trapped state.

In summary, if quantum interferences are taken into account, vacuum-induced optical bistability in \( V \)-type three-level atoms may be controlled by the parameters characterizing the squeezed field.

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