Light scattering by small cylinders

The light scattering that happens in a colloidal suspension is due to the interaction between the incident light and the suspension structures. Suppose a laser beam, with wavelength in the visible range, $\lambda_0$, that is impinging on a colloidal suspension characterized by a density of aggregates per unit volume, $n_0$. The incident electric field can be described as a wave traveling in the $Z$ axis (van de Hulst, 1981)

$$u_0 = e^{-ikz-i\omega t} \quad \text{being} \quad k = \frac{2\pi n_0}{\lambda_0}$$

where $k$ is the wave vector, and $n_0$ is the solvent refractive index. The wave scattered by each cluster at a point at a large distance $r$ from the sample can be written as an spherical wave with amplitude inversely proportional to the distance $r$. We may therefore write it in the form (van de Hulst, 1981)

$$u = S(\theta, \psi) e^{ikr+i\phi} = S(\theta, \psi) e^{ikr+i\phi} u_0$$

(3.3)

Therefore, defining the amplitude function of the scattering aggregate $S(\theta, \psi) = s(\theta, \psi)e^{i\phi(\theta, \psi)}$, this function describes not only the amplitude $s(\theta, \psi)$ but also the phase $\phi(\theta, \psi)$ of the scattered wave.

To solve the scattering problem of the suspension some approximations are needed. Firstly, we will suppose that the volume fraction is small so each aggregate is far enough from the others and so the scattering can be considered incoherent. Secondly, the sample thickness $c$ is so small that the multiple scattering contribution can be considered negligible. In this way, if we consider that an aggregate $j$ has an amplitude function $S(\theta, \psi)$, the intensity scattered by the colloidal suspension will be the sum of the intensities scattered by each aggregate $j$

$$I_{scatt}(\theta, \psi) = \sum_j I(\theta, \psi)$$

where

$$I(\theta, \psi) = \frac{2g^2(\theta, \psi)I_0}{k^2} I_0$$

(3.4)

(3.5)

where $I_0$ is the intensity of the incident light. In the MRS suspensions used in this dissertation, the structures responsible for the scattering are the chains induced in the medium when the magnetic field is applied. We will consider that these structures are, in a first approximation, cylinders with radius $a$ (the radius of the particle) and length $L$.

There is an analytical solution (van de Hulst, 1981) to the scattering problem of a cylinder which axis is parallel to the field propagation direction (following the $Z$ axis) (see figure 3.14). Calculating the scattering in the point at a large distance $r$ from the sample (so, $r \gg c$), $\lambda_0 \simeq 100 \, \mu m$, and $\lambda = 632.8 \, nm$), the scattering amplitude function that appears in equation (3.3) is given by (van de Hulst, 1981)

$$S(\theta, \psi) = \frac{kL}{r} \sin \left( \frac{kLr}{L} \right) T(\theta)$$

(3.6)

where $r = \frac{\omega}{c}$, $r T(\theta)$ is the scattering amplitude function for an incoherent cylinder with the form

$$T(\theta) = \sum_{j} \sum_{m} b_j \cos m \theta = b_0 + 2 \sum_{j} b_j \cos m \theta$$

(3.7)

$T(\theta)$ takes different values depending on the orientation of the electric field vector being parallel ($T(\theta)$), or perpendicular ($T'(\theta)$) to the cylinder axis. The coefficients of these two functions are defined by (van de Hulst, 1981):

$$d_0 = -mJ_{-1}(\eta n_0)J_0(ba) - J_1(\eta n_0)J_0(ba)$$

$$d_1 = mJ_{-1}(\eta n_0)J_1(ba) - J_1(\eta n_0)J_1(ba)$$

$$d_0 = -mJ_{-1}(\eta n_0)H_{0}(ba) - J_1(\eta n_0)H_{0}(ba)$$

$$d_1 = mJ_{-1}(\eta n_0)H_{1}(ba) - J_1(\eta n_0)H_{1}(ba)$$

(3.8)

where $m = n_0/c$, $J_0(x)$ is the first kind Bessel function, $J'_0(x)$ its derivative, $H_0(x)$ is the second kind Bessel function given by $H_0(x) = J_0(x) - \frac{\pi x}{2}$, where $J'_0(x)$ is the second kind Bessel function and $H'_0(x)$ its derivative. Using this result, the intensity taken in the scattering pattern is

$$I(\theta, \psi) = \frac{2g^2(\theta, \psi)I_0}{k^2} I_0$$

(3.9)

The scattering pattern for a cylinder is then different in the $\theta$ and $\psi$ directions. The variation of the intensity with $\theta$ has information about the cylinder width, while the variation of the intensity with $\psi$ has information about its length. Then, as an example, in the case of an infinite cylinder, the scattering pattern would be confined to the equatorial plane (XY plane). From the light intensity scattered on the XY plane at a distance $l$ far enough from the sample and for different angles with respect to the incident angle (that is, $\psi = 0, \phi = 0$), the chain length and the distance between cylinders could be obtained.
Optical anisotropy in a medium of small cylinders

In the previous section it has been shown how the scattering amplitude functions are different for the two polarisation components of the electric field (parallel and perpendicular to the axis of the cylinder (eqs. (3.16)–(3.18)). Therefore we are going to rewrite both, the incident wave (eq. (3.1)) and the scattered wave (eq. (3.3)) taking into account the two components of polarisation of light:

\[
\begin{pmatrix}
E_x^i \\
E_y^i
\end{pmatrix} = e^{-i k z \sin \phi} \begin{pmatrix}
E_x^i \\
E_y^i
\end{pmatrix} \\
\begin{pmatrix}
E_x^s \\
E_y^s
\end{pmatrix} = \frac{S_0(\phi, \psi)}{S_i(\phi, \psi)} \begin{pmatrix}
E_x^s \\
E_y^s
\end{pmatrix} e^{i k z \sin \phi} \begin{pmatrix}
E_x^i \\
E_y^i
\end{pmatrix}
\]

Here again, the scattering amplitude functions \( S_0(\phi, \psi) \) are complex functions. The amplitude of the total wave in the forward direction, that is, along the Z axis, it can be written adding the amplitudes of the incident and the scattered wave from all the aggregates. Doing \( \phi = 0 \) and \( \psi = 0 \) in equation (3.13) we can write the amplitude of the scattered wave by a cylinder of length \( L \) as

\[
\begin{pmatrix}
E_x^s(0) \\
E_y^s(0)
\end{pmatrix} = \frac{1}{1 - i 2 \frac{n_1 - n_2}{k} L} \begin{pmatrix}
E_x^i(0) \\
E_y^i(0)
\end{pmatrix}
\]

where \( E_i(0) \) and \( E_s(0) \) are the scattering amplitude functions defined in eq. (3.17) and they do not depend on \( L \). The total wave corresponding to a sample with thickness \( c \) which contains \( n_c \) cylinders of length \( L \) per unit volume is (van de Hulst, 1981)

\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix} = \begin{pmatrix}
1 - i 2 \frac{n_1 - n_2}{k} L \\
1 - i 2 \frac{n_1 - n_2}{k} L
\end{pmatrix} \begin{pmatrix}
E_x(0) \\
E_y(0)
\end{pmatrix}
\]

One can observe in equations (3.10)-(3.17) that both components of polarisation of the field (parallel and perpendicular to the axis of the chain) suffer a change as in the phase (dispersion) as in the amplitude (attenuation), and, what is more important, this change is different for each polarization component.

The result of equations (3.10)-(3.17) can be represented in a formal manner as the propagation effect of the incident wave through a medium with a complex index of refraction \( n \). This index will be different for each polarisation direction of the electric field. We will call \( n_\parallel = n'_\parallel - i n'_\parallel \) to the index in the polarization direction corresponding to the axis of the chains, and \( n_\perp = n'_\perp - i n'_\perp \) to the one in the polarization direction perpendicular to the axis of the chains. This representation allows us to calculate the relative change of the wave amplitude when crossing the medium for each one of the two polarization components. Then we can compare this change with equations (3.10)-(3.17). For the parallel and perpendicular components to the chains axis, respectively, we have (van de Hulst, 1981)

\[
e^{-i \phi_L} = 1 - i \frac{n'_\parallel - 1}{n'_\parallel} = 1 - \frac{2 n_1 \sigma L}{k} T'(0)
\]

\[
e^{-i \phi_L} = 1 - i \frac{n'_\perp - 1}{n'_\perp} = 1 - \frac{2 n_1 \sigma L}{k} T''(0)
\]

Starting from these expressions the complex index of refraction of the medium can be calculated for each polarization component

\[
\begin{align*}
n_\parallel &= n'_\parallel - i n'_\parallel = n_\parallel \left[ 1 - \frac{2 n_1 \sigma L}{k} T'(0) \right] \\
n_\perp &= n'_\perp - i n'_\perp = n_\perp \left[ 1 - \frac{2 n_1 \sigma L}{k} T''(0) \right]
\end{align*}
\]

This optical anisotropy induced in the medium due to the formation of chains aligned with the field leads to phenomena of birefringence (different phase velocity for each polarization component) and dichroism (different attenuation for each polarization component). Using the previous expressions (3.20) and (3.21), we can calculate the birefringence \( \Delta n' \) and the dichroism \( \Delta n'' \) which are defined as the anisotropies of the real and imaginary parts, respectively of the refractive index:

\[
\Delta n' \equiv n'_\parallel - n'_\perp = \frac{2 n_1 \sigma L}{k} \left[ \text{Im} \left[ T'(0) \right] - \text{Im} \left[ T''(0) \right] \right]
\]

\[
\Delta n'' \equiv n''_\parallel - n''_\perp = \frac{2 n_1 \sigma L}{k} \left[ \text{Re} \left[ T'(0) \right] - \text{Re} \left[ T''(0) \right] \right]
\]

Therefore, the anisotropy on the shape of the structures induced in the suspension makes the scattered light different depending on the polarization of the incident light. Hence, this type of optical anisotropy is called form anisotropy. It is important to point out that spherical particles consist of an optically isotropic material have not dichroism. In general, when the length scale of the scattering objects is comparable or larger than the laser wavelength, like in our case, the induced dichroism will be larger than the induced birefringence (van de Hulst, 1981).

In figure 3.16 we present the birefringence \( \Delta n' \) and the dichroism \( \Delta n'' \) calculated from expressions (3.22) and (3.23) for an aqueous suspension with \( n_\parallel = 1.33 \) and \( n_\perp = 1.32 \) (van de Hulst, 1981).

In our experiments the chains induced by the field are one particle thick, that is, the cylinders radius will vary between 0.32 \( \mu \text{m} \) and 0.62 \( \mu \text{m} \).
Scattering dichroism setup

The most simple optical setup to measure dichroism is one where two polarizers, the sample and a photodiode, however, with this system we can not separate the contribution of the extinction (eq. (5.6)) and the contribution of the orientation angle of the structures to the measured intensity. Therefore, it is necessary to use more sophisticated methods that allow us to measure dichroism in nanoseconds, as the modulation polarization method. There exist different polarization modulation methods, the one we will use is based on the modulation of the phase of an optical element where polarization axis is fixed. This one is called field-effect modulation method since it is the application of an external field onto the optical element, the case that will produce the modulation of the polarization of the light transmitted by the device.

An schematic diagram of the optical setup used to measure linear scattering dichroism (LU) is shown in figure 3.17. A photo of the real system is shown in figure 3.18. The monochromatic light that produces an $\delta_{\text{ PEM}}$ ($\delta_{\text{ PEM}}$ = 92.8 nm, power 0.5 mW) pass through a polarizer whose axis is at 0° (this will be reference axis of our optical system). The light that comes out linearly polarized, it goes then to a $\delta_{\text{ PEM}}$ PEM (photomultiplier modulator) placed at 45° respect to the incident polarizer. The PEM induces a sinusoidal time-dependent retardation between the two components of polarization of the light. This retardation can be written as $\delta_{\text{ PEM}} = \delta_{\text{ PEM}}(t) = A sin(\omega_{\text{ PEM}} t)$, where $A$ is the modulation amplitude, $f_{\text{ PEM}} = \omega_{\text{ PEM}}/2\pi = 50$ kHz is the modulation frequency. The light that leaves the PEM pass through a quarter wave plate at 90° angular displacement with respect to the polarizer. Then, the laser beam traverses the sample cell and the transmitted light is detected by a UDT Sensors 8212 photodiode.

A detailed calculation of the output intensity using Jones matrices has been carried out in the Appendix B (Filster, 1995). As a result, we get

$$
I = \frac{I_0}{2} [cos\theta'' - \frac{-2f_0(A)(\sin(2\theta')\sin(\varphi'\omega_{\text{ PEM}}))}{2f_0(A)(\cos(2\theta')\cos(2\varphi'\omega_{\text{ PEM}}))}]
$$

where $I_0$ is the incident light intensity, $\sigma''$ is the extinction introduced by the sample, $A, \varphi$ are linearly polarized functions and $\theta''$ is the orientation angle of the structures respect to the axis of the polarizer.

This signal is sent to two UDT-18A, Protection Applied Research lock-in amplifiers and to a photodiode-amplifier. The first lock-in extracts the component of the signal that oscillates with the PEM frequency ($\omega_{\text{ PEM}}$) and the second one extracts the component that oscillates with the second harmonic ($2\omega_{\text{ PEM}}$). The photodiode-amplifier is also provided by a low-pass filter that extracts the DC component of the signal ($I_0$). Therefore, we are going to measure three signals:

$$
I_{\text{ DC}} = \frac{I_0}{2} \sin\theta''
$$
$$
I_{\omega_{\text{ PEM}}} = -I_0 A_0 (\sin(2\theta')\sin(\varphi'\omega_{\text{ PEM}}))
$$
$$
I_{2\omega_{\text{ PEM}}} = -I_0 A_0 (\sin(2\theta')\cos(2\varphi'\omega_{\text{ PEM}}))
$$

The three voltages ($I_{\text{ DC}}, I_{\omega_{\text{ PEM}}}$ and $I_{2\omega_{\text{ PEM}}}$) are digitized using a 16-bit resolution A/D data acquisition board (National Instruments PCI-MIO-16XE-16) with 8 simultaneous sampling channels and sampling frequency of 100 kHz.

EXTINCTION
$$
\delta'' = \frac{2\pi \Delta n'' e}{\lambda
$$

DICROISM
$$
\Delta n'' = n''' - n''''
$$

EXTINCTION ANGLE $\equiv \theta''$

\[n'''' \quad \perp \]

\[X \quad \perp n'''' \quad \parallel \]

\[Y \quad \parallel n'''' \quad \perp \]

\[\theta''\]

\[\lambda / 4 (0°)\]

\[P (0°)\]

\[Laser\]

\[\text{Sample cell}\]

\[\text{Photodiode}\]

\[\text{DCI}\]

\[\text{PEM controller}\]

\[\text{Photodiode amplifier}\]

\[\text{Filter}\]

\[\text{Lock-in } 1f_{\text{ PEM}}\]

\[\text{Lock-in } 2f_{\text{ PEM}}\]
Appendix B

Intensity measured on scattering dichroism experiments

When light transmits through an optical system changes in its polarization can be induced. An easy mathematical way of understanding such change is to represent the polarized light as Jones matrices. In this Appendix we calculate in detail the output intensity form the linear dichroism system described in section 3.3.3 (Fulcher, 1995).

A transversal plane wave of monochromatic light, coherent and perfectly polarized that propagates in vacuum along the z axis is written by

\[ \vec{E}(z,t) = E_0 e^{i(k_0 z - \omega t + \delta_0)} \hat{\vec{z}} \]  

\[ \vec{E}'(z,t) = E_0 e^{i(k_0 z - \omega t + \delta_0)} \hat{\vec{z}} \]  

where \( E_0 \) and \( E_0' \) are the amplitudes, \( k_0 = 2\pi / \lambda \) is the wave vector, \( \lambda \) is the wavelength of the incident light, and \( \delta_0, \delta_0' \) are the appropriate phases. Due to the localization of the electric vectorial field normal to the axis of the wave propagation, it is convenient to write it as:

\[ \vec{E} = \begin{pmatrix} E_0(t) \\ E_0'(t) \end{pmatrix} = \begin{pmatrix} E_0 e^{i\delta_0} \\ E_0' e^{i\delta_0'} \end{pmatrix} e^{i(k_0 z - \omega t)} = e^{-i\delta} e^{i(k_0 z - \omega t)} \begin{pmatrix} E_0 \\ E_0' \end{pmatrix} e^{i\delta} \]  

where \( \delta = \delta_0 - \delta_0' \) is the phase difference between both components and the intensity can be written as \( I = |E_0|^2 + |E_0'|^2 \). Hence, the polarization state of the electrical

\[ \text{field vector can be represented as a column matrix (named Jones vector):} \]

\[ \textbf{J} = \begin{pmatrix} E_0 \\ E_0' e^{i\delta} \end{pmatrix} \]  

The interaction between the electric field and an anisotropic medium which complex refractive index is \( n = n' + i n'' \) can make its orthogonal polarization components to have different phases (this means that a delay can be produced \( \delta \) between the phase of one polarization component and the other) and different attenuation (inducing a different extinction \( \delta'' \) in one component with respect the other). This retardance can be measured by means of the induced birefringence in the medium and the extinction through the dichroism. Both magnitudes are related as follows:

\[ \text{Retardance: } \delta'' \Rightarrow \text{Birefringence: } \Delta n'' \equiv n'' - n''_0 = \frac{\lambda \delta''}{2\pi c} \]  

\[ \text{Extinction: } \delta'' \Rightarrow \text{Dichroism: } \Delta n'' \equiv n'' - n''_0 \]  

where \( c \) is the sample thickness.

The optical components that will be part of the system are different depending on the anisotropy (birefringence or dichroism) that will be measured. When the anisotropic properties of the sample are known a priori, the optical device that has to be used is the one used for measuring birefringence\(^3\) so in this case, the intensity will have contributions of both anisotropies (Johnson and Fuller, 1983). In order to discern between the contribution due to birefringence and the one due to dichroism is necessary to use a device able to measure dichroism. In our case, in principle, birefringence of the sample was measured finding out that its value was negligible related to the value of dichroism.

As seen in section 3.3.3 the optical device used for the dichroism measurements (Fulcher, 1995) is formed by a linear polarizer oriented at 0°, a photelastic modulator at 45°, a retarder plate /4 at 90° and the sample which polarization axis form an angle \( \theta'' \) with the axis of the polarizer (see fig. B.3).

\(^3\) The simplest optical device in this case would be formed by two crossed polarizers, one before and other after the sample.
B.1 Jones matrices of the optical elements

Using the Jones matrix representation, we can calculate the emerging vector corresponding to the transmitted wave through the different optical elements forming the system by computing the product of the matrix of each optical element (matrices $2 \times 2$) with the Jones vector of the state of polarization. This way, the output polarization vector becomes

$$
\begin{pmatrix}
E_x'
\end{pmatrix} = J_{pm}(\theta') \cdot J_{pm}(\theta) \cdot J_{pd}(45') \cdot 
\begin{pmatrix}
E_x
\end{pmatrix}
$$

(B.7)

### B.1 Jones matrices of the optical elements

#### Linear polarizer

When light of intensity $I_0$ coming from a non-polarized source reaches a linear polarizer with a transmission axis oriented at $\theta'$, it emerges polarized in that direction so that the outgoing Jones vector can be written as\(^2\):

$$
\begin{pmatrix}
E_x'
E_y'
\end{pmatrix} = \sqrt{\frac{I_0}{2}} 
\begin{pmatrix}
1
0
\end{pmatrix}
$$

(B.8)

\(^2\)The $1/2$ factor comes from the average square cosine, because the polarization angle of the light coming from the source is random.

#### Photoelastic modulator (PEM)

The photoelastic modulator (PEM) introduces a retardation $\delta_{PEM} = \lambda \sin(\omega_{PEM} t)$. In our system, the modulator is oriented at $45^\circ$ so can be represented by the following matrix:

$$
J_{PEM}(45') = \begin{pmatrix}
\cos\left(\frac{\delta_{PEM}}{2}\right) & i \sin\left(\frac{\delta_{PEM}}{2}\right) \\
\sin\left(\frac{\delta_{PEM}}{2}\right) & \cos\left(\frac{\delta_{PEM}}{2}\right)
\end{pmatrix}
$$

(B.9)

Because the retardation introduced by the PEM is sinusoidal, the Jones vector at the output of the PEM will contain components of the form $\cos\delta_{PEM} = \cos(\lambda \sin(\omega_{PEM} t))$ and $\sin\delta_{PEM} = \sin(\lambda \sin(\omega_{PEM} t))$. Consequently, the temporal response is rather complicated since the light polarization leaving the PEM varies with time changing from circular to elliptical and then linear (see lower part of fig. B.1):

$$
\begin{pmatrix}
E'_x
E'_y
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
\cos\left(\frac{\delta_{PEM}}{2}\right) \\
\sin\left(\frac{\delta_{PEM}}{2}\right)
\end{pmatrix}
$$

(B.10)

#### Quarter-wave plate

The matrix of a quarter-wave plate ($J_{qwp}$) oriented at $0^\circ$ has the following form

$$
J_{qwp}(\theta') = \frac{1}{\sqrt{2}} 
\begin{pmatrix}
1+i & 0 \\
0 & 1-i
\end{pmatrix}
$$

(B.11)

Leaving the $\lambda/4$ we will obtain linear polarized light at an angle $\delta_{PEM}/2$, with its polarization plane changing periodically with a frequency $\omega_{PEM}$:

$$
\begin{pmatrix}
E'_x
E'_y
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
1+i \\
0
\end{pmatrix}
$$

(B.12)

If we had not included the $\lambda/4$ in our optical system, we would not be able to determine simultaneously the value of the dichroism induced by the sample, $\delta'$ and the orientation angle of the structures $\theta'$, because the intensity we would obtain would not be able to be factorized.

#### Sample

In order to calculate the output intensity of the system we need to represent the polarization effects of the sample on the incident light. In general, the sample is
a medium that presents dichroism $\Delta n^d$ and birefringence $\Delta n^i$. Nevertheless, the birefringence obtained in the experiments is much smaller than dichroism, so it can be neglected, being the Jones Matrix of the sample:

$$J_n(\theta^d) = \begin{pmatrix} \cosh(\xi_{2d}) & -\cos(2\theta^d)\sinh(\xi_{2d}) \\ -\sin(2\theta^d)\sinh(\xi_{2d}) & \cosh(\xi_{2d}) + \cos(2\theta^d)\sinh(\xi_{2d}) \end{pmatrix}$$ (B.13)

where $\theta^d$ represents the extinction introduced by the sample already defined in equation (B.6). At the sample output the polarization vector will be:

$$\begin{pmatrix} E_x^{\text{out}} \\ E_y^{\text{out}} \end{pmatrix} = \sqrt{\frac{I_0}{2}(1+i)} \begin{pmatrix} c\chi_{2d}(\xi_{2d}) - c(2\theta^d)\sinh(\xi_{2d}) + \sin(2\theta^d)\sinh(\xi_{2d}) \\ s\chi_{2d}(\xi_{2d}) + c(2\theta^d)\sinh(\xi_{2d}) - \sin(2\theta^d)\sinh(\xi_{2d}) \end{pmatrix}$$ (B.14)

where the following notation: $c = \cos$, $s = \sin$, $\chi = \cosh$ and $\sinh$ has been used.

### B.2 Output intensity of the system

Finally, we calculate the output intensity $I$ from the polarization vector. The intensity is defined as the sum of the modulus-squared of the polarization vector components, that is, $I = |E_x^{\text{out}}|^2 + |E_y^{\text{out}}|^2$. Therefore, the output intensity will be:

$$I = I_0 \left[ \cosh(\theta^d) - \sin(2\theta^d)\sinh(\theta^d) \right]$$ (B.15)

Because the retardation $\phi_{PEM}$ is sinusoidal, the temporal behavior of the intensity has components of the form $\sin(\omega_{PEM}t)$ and $\cos(\omega_{PEM}t)$. The information to be obtained from the intensity is the value of the coefficients $\sin(\phi_{PEM})$ and $\cos(\phi_{PEM})$ so we can analyze the Fourier content of the signal using the following expressions:

$$\begin{align*}
\sin(\phi_{PEM}) &= 2 \sum J_m(\lambda) \sin(2\pi m \omega_{PEM} t) \\
\cos(\phi_{PEM}) &= J_0(\lambda) + 2 \sum J_m(\lambda) \cos(2\pi m \omega_{PEM} t)
\end{align*}$$ (B.16, B.17)

where $J_m(\lambda)$ is the Bessel function of $m$ order and depends on the amplitude of the modulation introduced by the PEM, $\lambda$. This value is obtained by simple calibration.

### Procedure

In our case, in order to simplify the expressions, we adjust the magnitude of the modulation so that $J_m(\lambda) = 0$. Ideally, this occurs at the first zero of the Bessel function, so that $\lambda = 2\pi$. Hence, the final expression for the intensity is:

$$I = I_0 \left[ \cosh(\theta^d) \\ -2J_0(2\pi)\sin(2\pi)\sinh(\theta^d) \right]$$ (B.18)

In order to perform the Fourier analysis of the digitalized signal, the signal detected by the photodiode (eq. (B.18)) is sent to two lock-in amplifiers (which extract the harmonic content of the signal). The continuous component, $I_{DC}$, can be extracted using a low-pass filter. It is necessary to outline that with the design used, even though the sample shows linear birefringence, its contribution will not show up in the final intensity.